Electron Spin Decoherence in Isotope-Enriched Silicon

Wayne Witzel, Malcolm Carroll, Andrea Morello,
Lukasz Cywinski, Sankar Das Sarma

arXiv:1008.2382

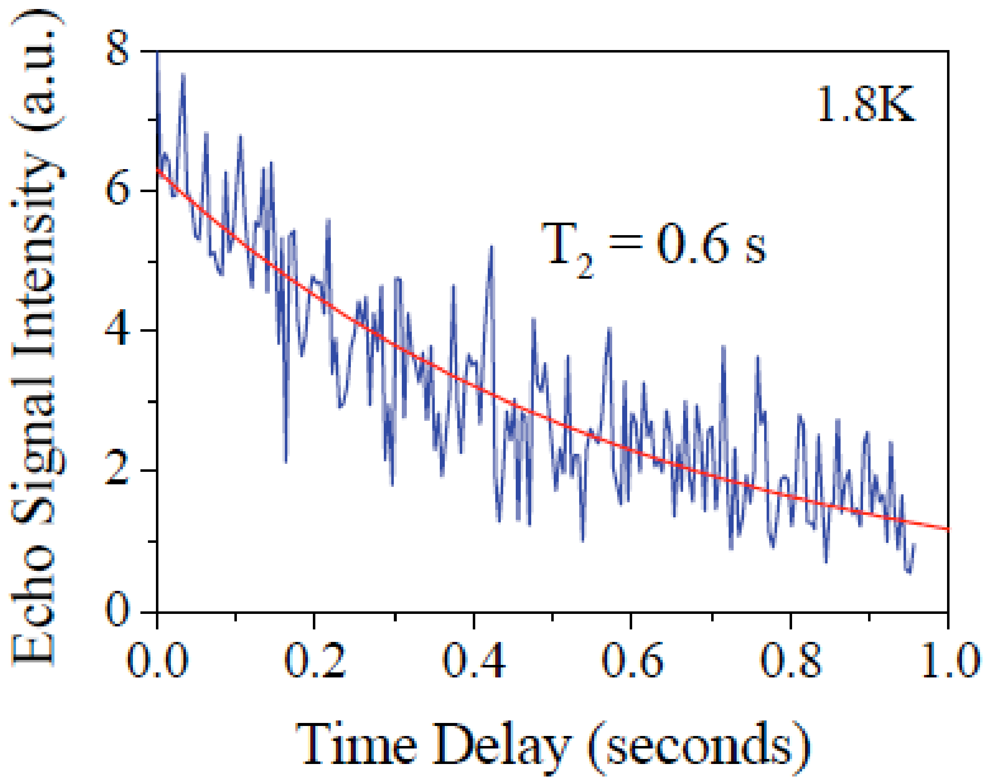
Silicon Qubit Workshop
Albuquerque, NM

This work was supported by the Laboratory Directed Research and Development program at Sandia National Laboratories.

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND2010-5774C

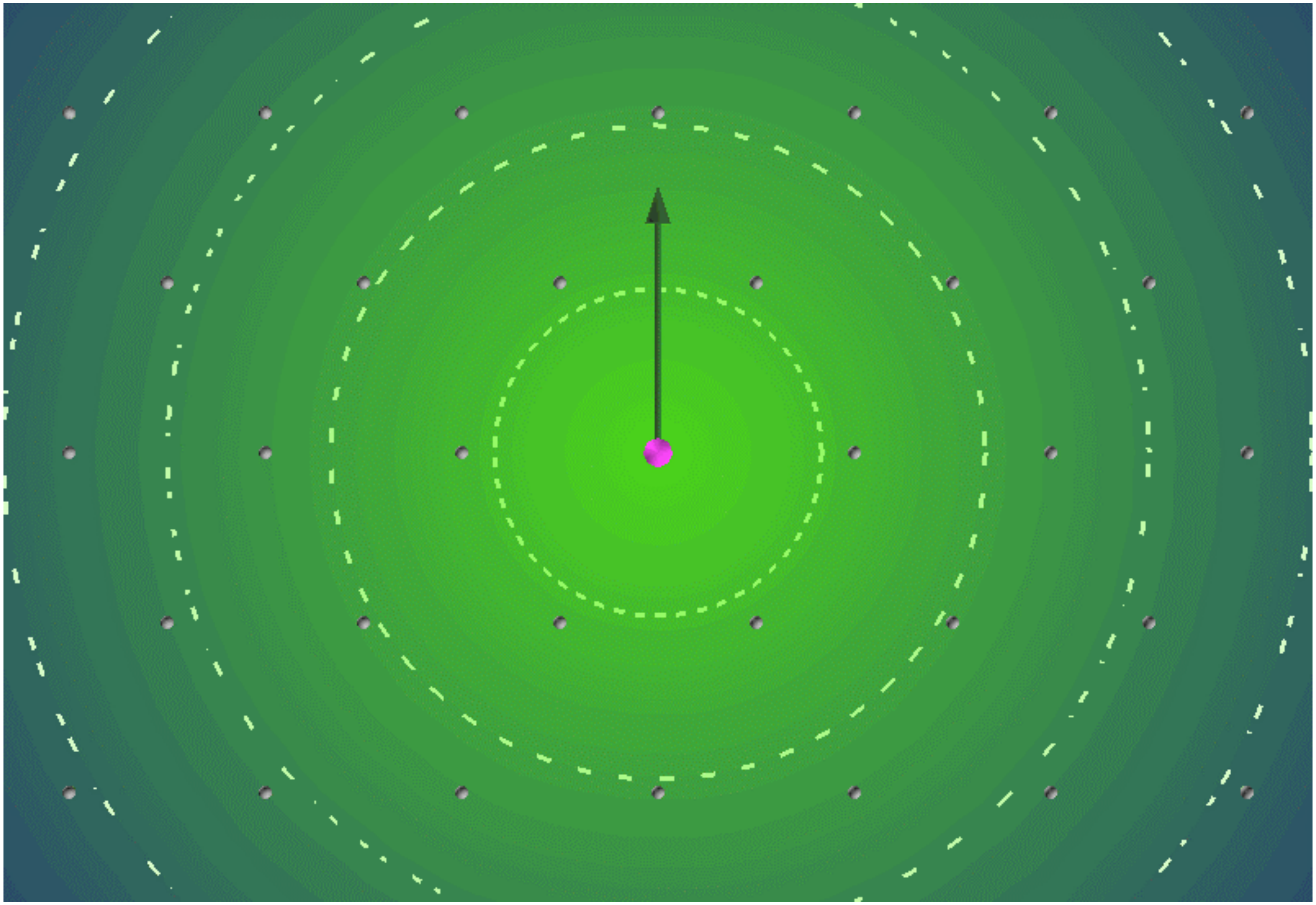
Experiments show T_2=600 ms. What limits this?

Donors: $1.2\times10^{14}/cm^3$, ^{29}Si: $2.5\times10^{18}/cm^3$ (~50 ppm)

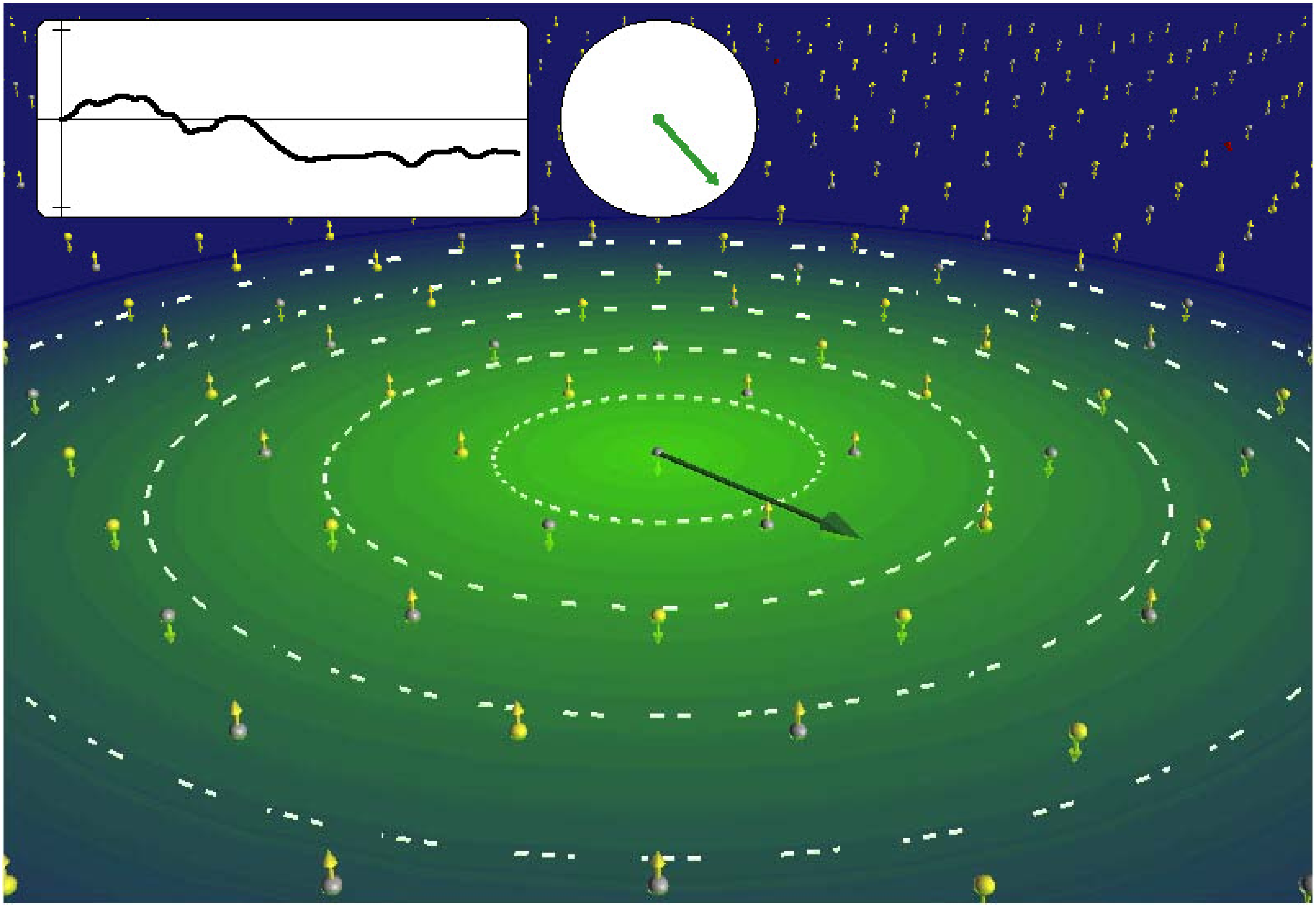


A. M. Tyryshkin et al, Silicon Qubit Workshop, August 2009, Berkeley, CA, USA

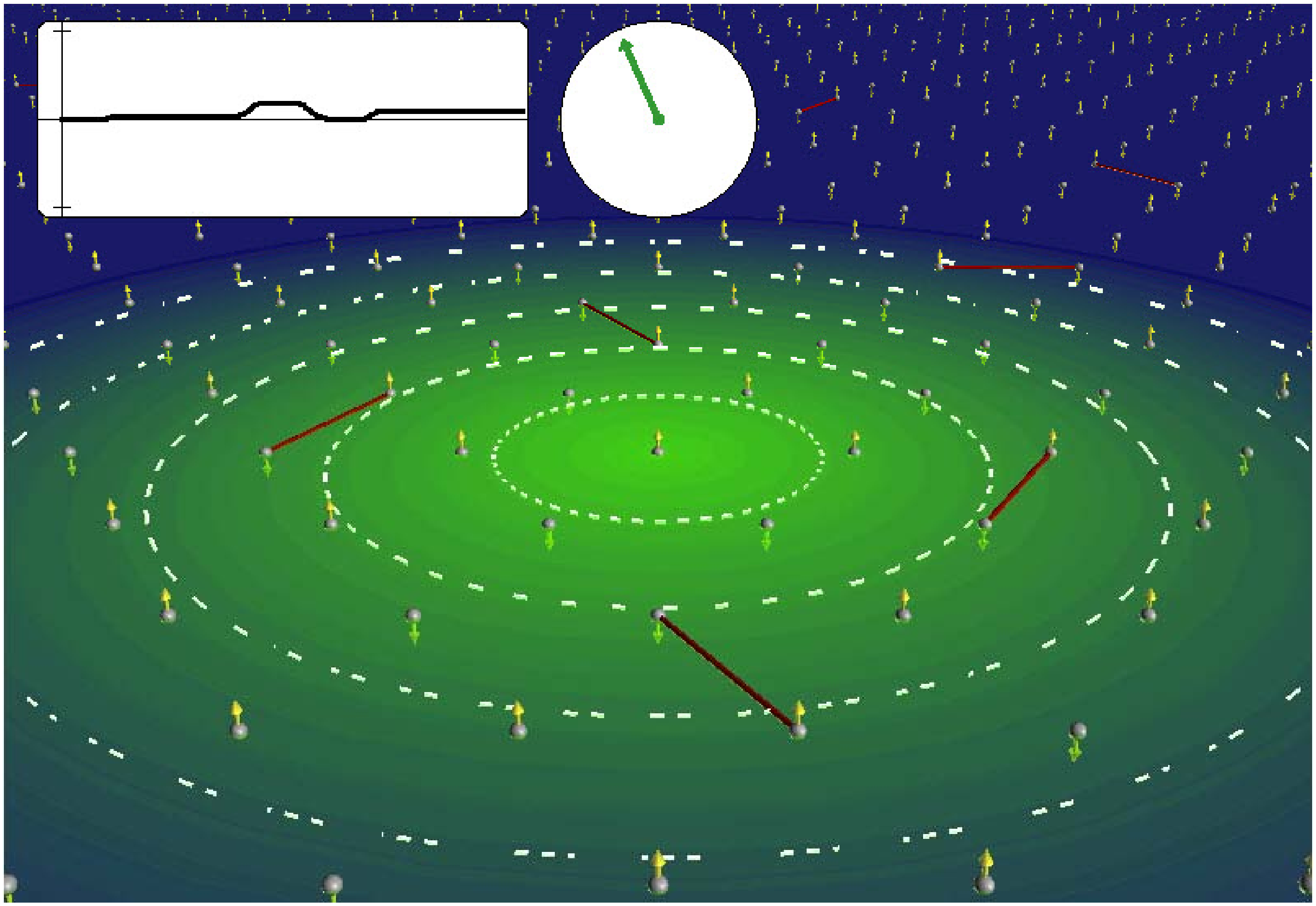
Central Spin Decoherence Problem

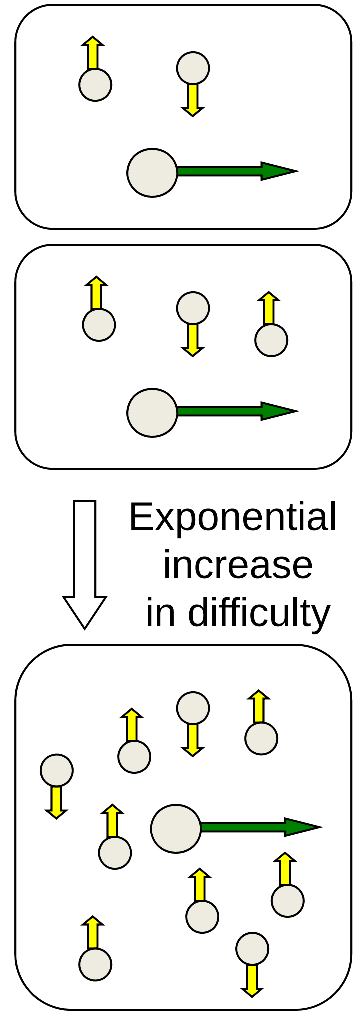


Central Spin Decoherence Problem



Central Spin Decoherence Problem

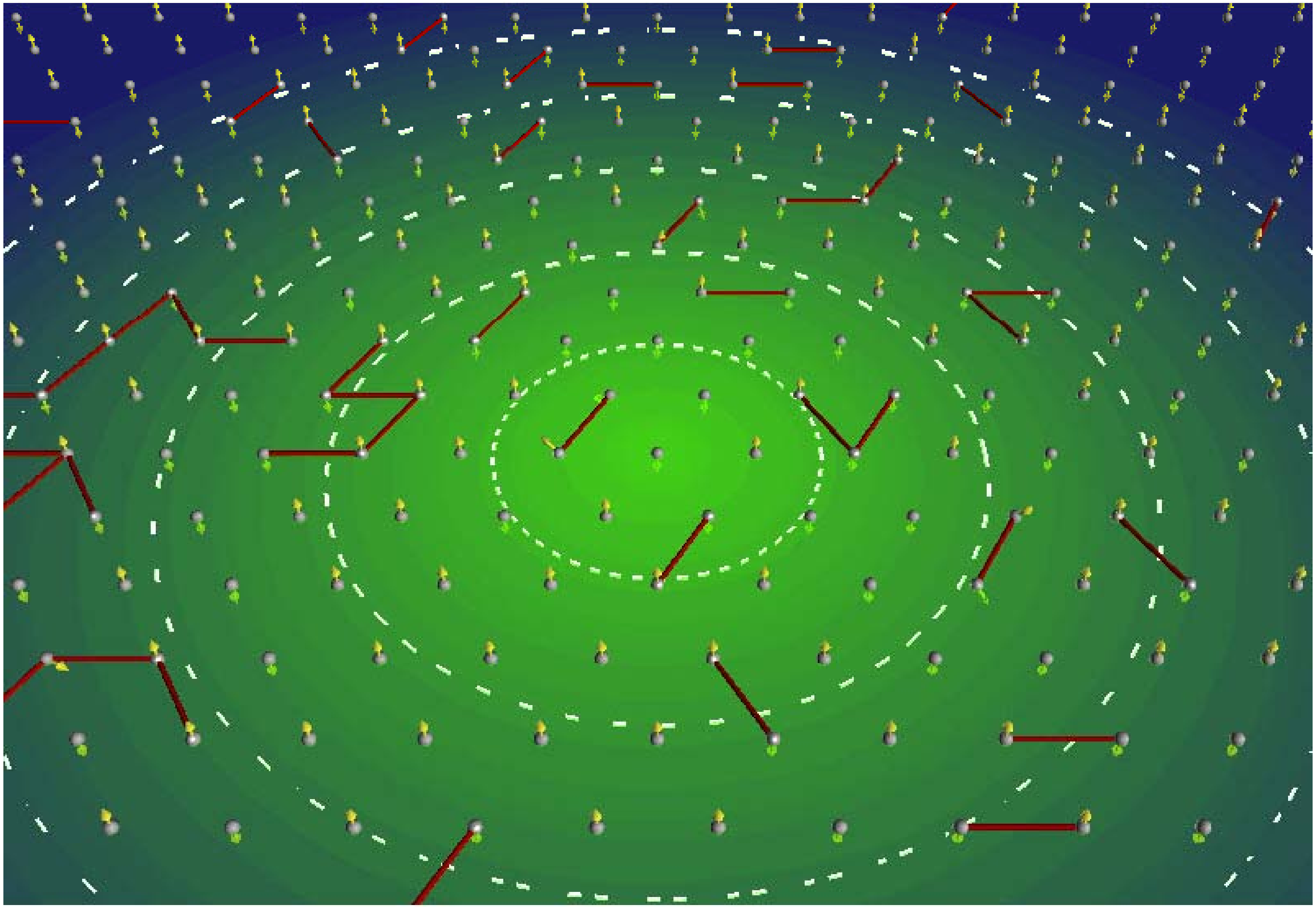




Spin Echo: decompressor are needed to see this picture.

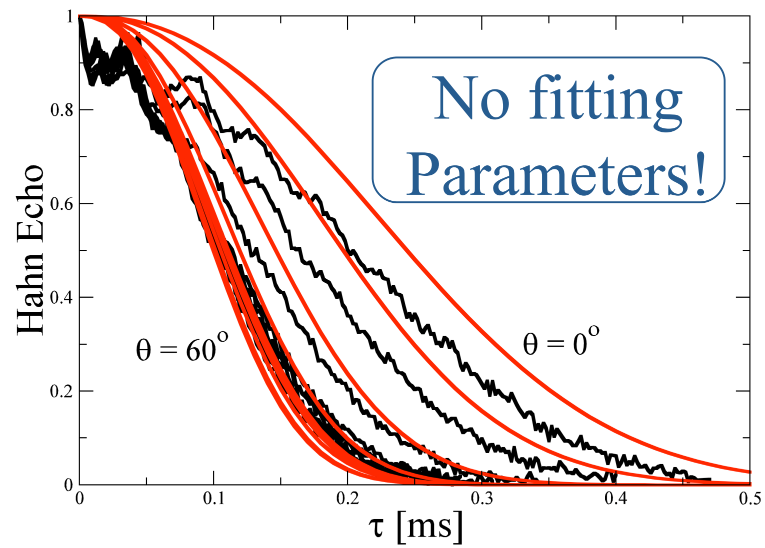
$$\rho(t) = \hat{U}\rho(0)\hat{U}^{\dagger}, \quad \rho_q(t) = \mathrm{Tr}_B\,\rho(t)$$

Visualizing Clusters



Cluster Expansions Provide a Solution

Cluster Expansion Method[1]



Experiment:

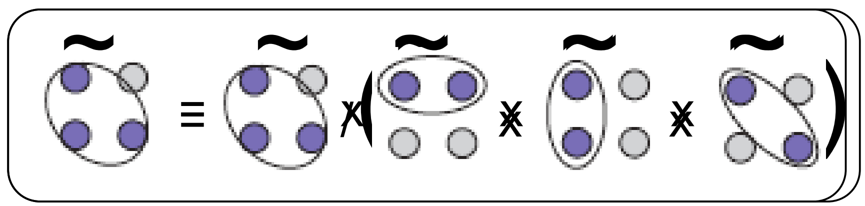
A.M. Tyryshkin, J.J.L. Morton, S.C. Benjamin, A. Ardavan, G.A.D. Briggs, J.W. Ager, S.A. Lyon, J. Phys.: Condens. Matter **18**, S783 (2006).

Cluster correlation expansion[2]:

$$L = \rho_q^{+-}(t)/\rho_q^{+-}(0)$$

$L_{\mathcal{S}} = L$ with the exclusion of spins outside of set $\mathcal{S}$

$$L = \prod_{\mathcal{S}} \tilde{L}_{\mathcal{S}}, \quad \tilde{L}_{\mathcal{S}} = L_{\mathcal{S}} / \prod_{\mathcal{C}\subset\mathcal{S}} \tilde{L}_{\mathcal{C}}$$



$$L_{\mathrm{CCE}}^{(k)} = \prod_{||\mathcal{S}||\leq k} \tilde{L}_{\mathcal{S}}.$$

[1] W. M. Witzel, Rogerio de Sousa, S. Das Sarma, Phys. Rev. B **72**, 161306(R) (2005); W. M. Witzel, S. Das Sarma, Phys. Rev. B **74**, 035322 (2006).

[2] Wen Yang, Ren-bao Liu, Phys. Rev. B **78**, 085315 (2008).

Slide 7

Problem formulation: Hamiltonian and evolution operator

$$\hat{H}_E = \sum_{i>j} \gamma_E^2 d(\mathbf{R}_i - \mathbf{R}_j)[\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+ - 4\hat{S}_i^z \hat{S}_j^z],$$

$$\hat{H}_N = \sum_{n>m} \gamma_N^2 d(\mathbf{r}_n - \mathbf{r}_m)[\hat{I}_n^+ \hat{I}_m^- + \hat{I}_n^- \hat{I}_m^+ - 4\hat{I}_n^z \hat{I}_m^z],$$

$$d(\mathbf{r}) = [1 - 3(r_z/r)^2]/4r^3$$

$$\hat{H}_{E-N} = \sum_{i,n} \gamma_E \gamma_N h_i(\mathbf{R}_i - \mathbf{r}_n)\hat{S}_i^z \hat{I}_n^z,$$

$$h_i(\mathbf{R}) = \frac{8\pi}{3}|\Psi_i(\mathbf{R})|^2 - \int d^3r |\Psi_i(\mathbf{r})|^2 \frac{|\mathbf{r} - \mathbf{R}|^2 - 3[r_z - R_z]^2}{|\mathbf{r} - \mathbf{R}|^5}$$

Spin Echo: $U(\tau) = \mathrm{e}^{-i\mathcal{H}\tau}\sigma_{x,e}\mathrm{e}^{-i\mathcal{H}\tau}.$

$$\rho(t) = \hat{U}\rho(0)\hat{U}^\dagger, \qquad \rho_q(t) = \mathrm{Tr}_B\, \rho(t)$$

Sparse Electron Spin Systems Present New Challenges

We find that we must include Ising interaction to spins outside of a cluster and average over their spin states in the evaluation for a given cluster.

Set of spin states

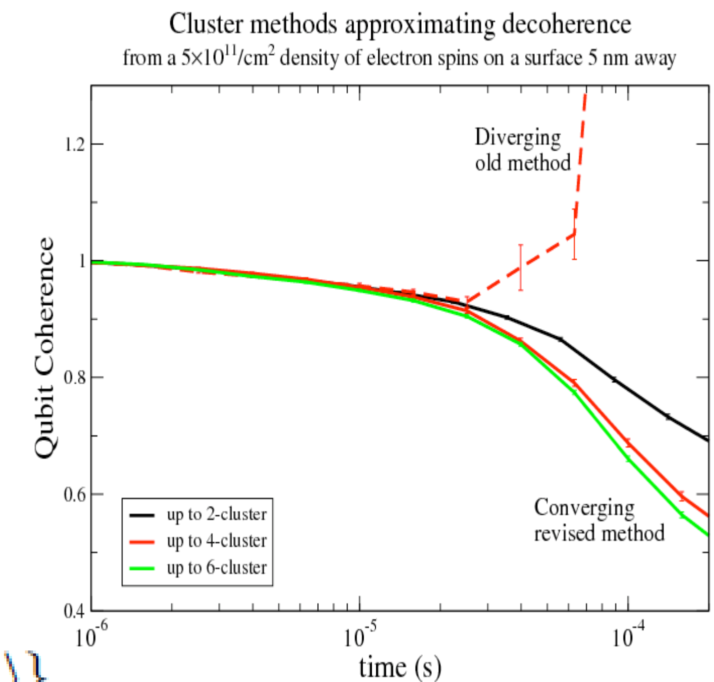
Spin state

$$\tilde{L}_{\mathcal{C}}^{\mathcal{K}} = \langle L_{\mathcal{C}}^{K} \rangle_{K\in\mathcal{K}} / \prod_{\mathcal{C}''\subset\mathcal{C}} \tilde{L}_{\mathcal{C}''}^{\mathcal{K}}$$

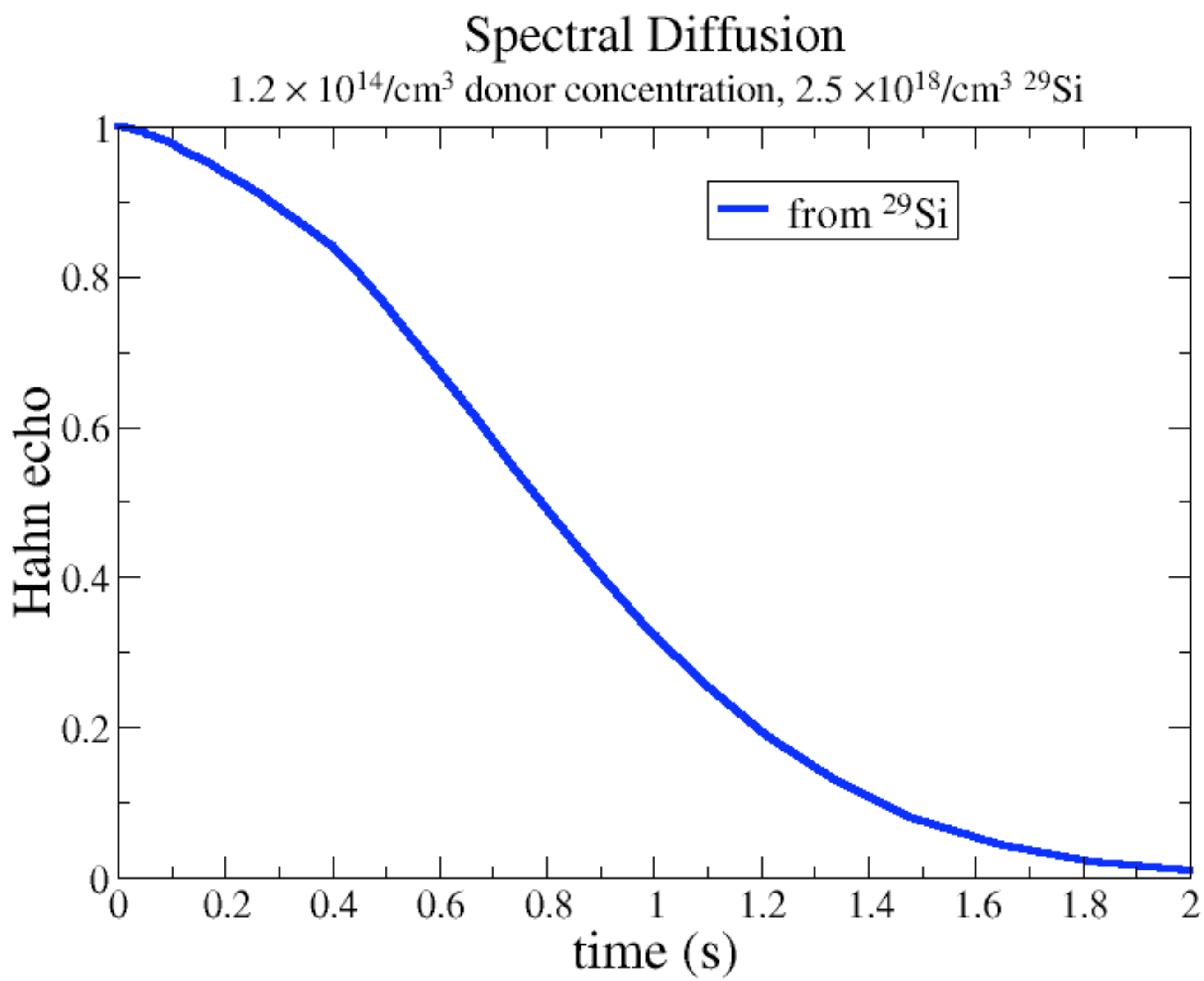
$$\mathcal{L}_{\Gamma}^{J} = \prod_{\mathcal{C}\in\Gamma} \tilde{L}_{\mathcal{C}}^{\mathcal{K}(J,\mathcal{C},\Gamma)}$$

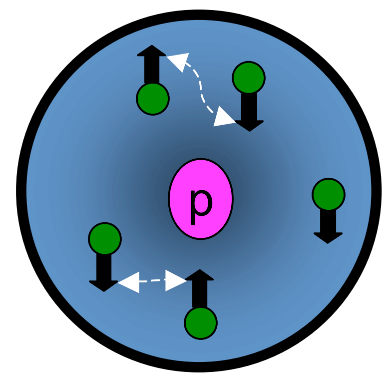
$$\mathcal{D}\left(\bigotimes_n |j_n\rangle, \bigotimes_n |j'_n\rangle\right) = \{n \mid |j_n\rangle \neq |j'_n\rangle\}$$

$$\mathcal{K}(J,\mathcal{C},\Gamma) = \{J' \mid \exists \mathcal{C}' \in \Gamma, \mathcal{C}' \supseteq \mathcal{C}, \mathcal{D}(|J\rangle,|J'\rangle) \subseteq \mathcal{C}'\}$$

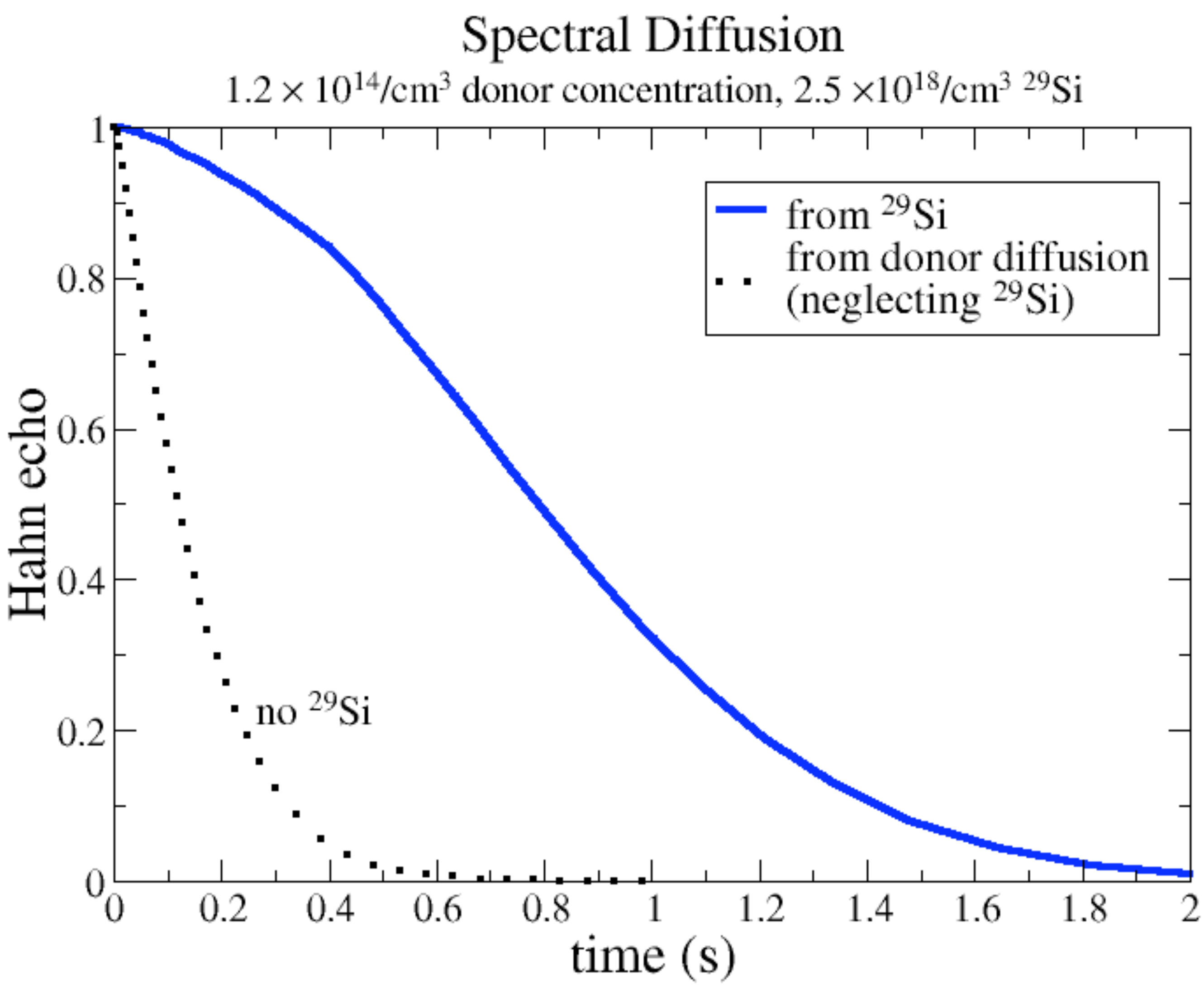


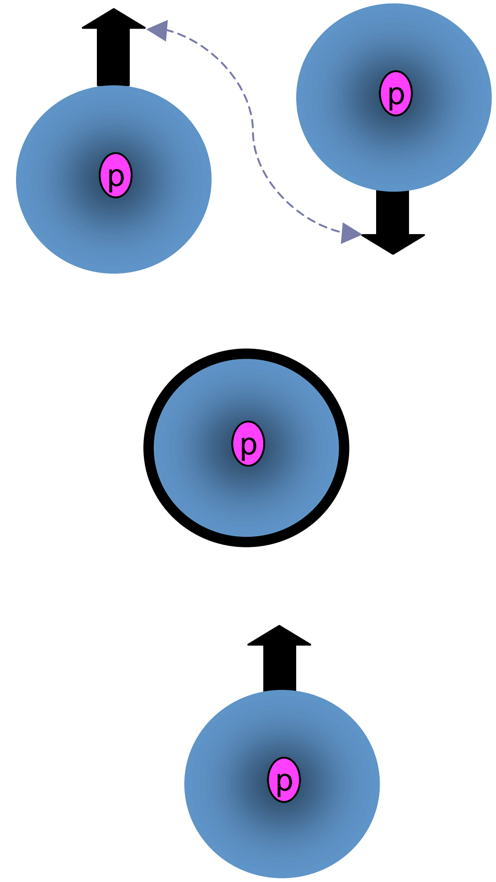
Theory: ^{29}Si spin noise



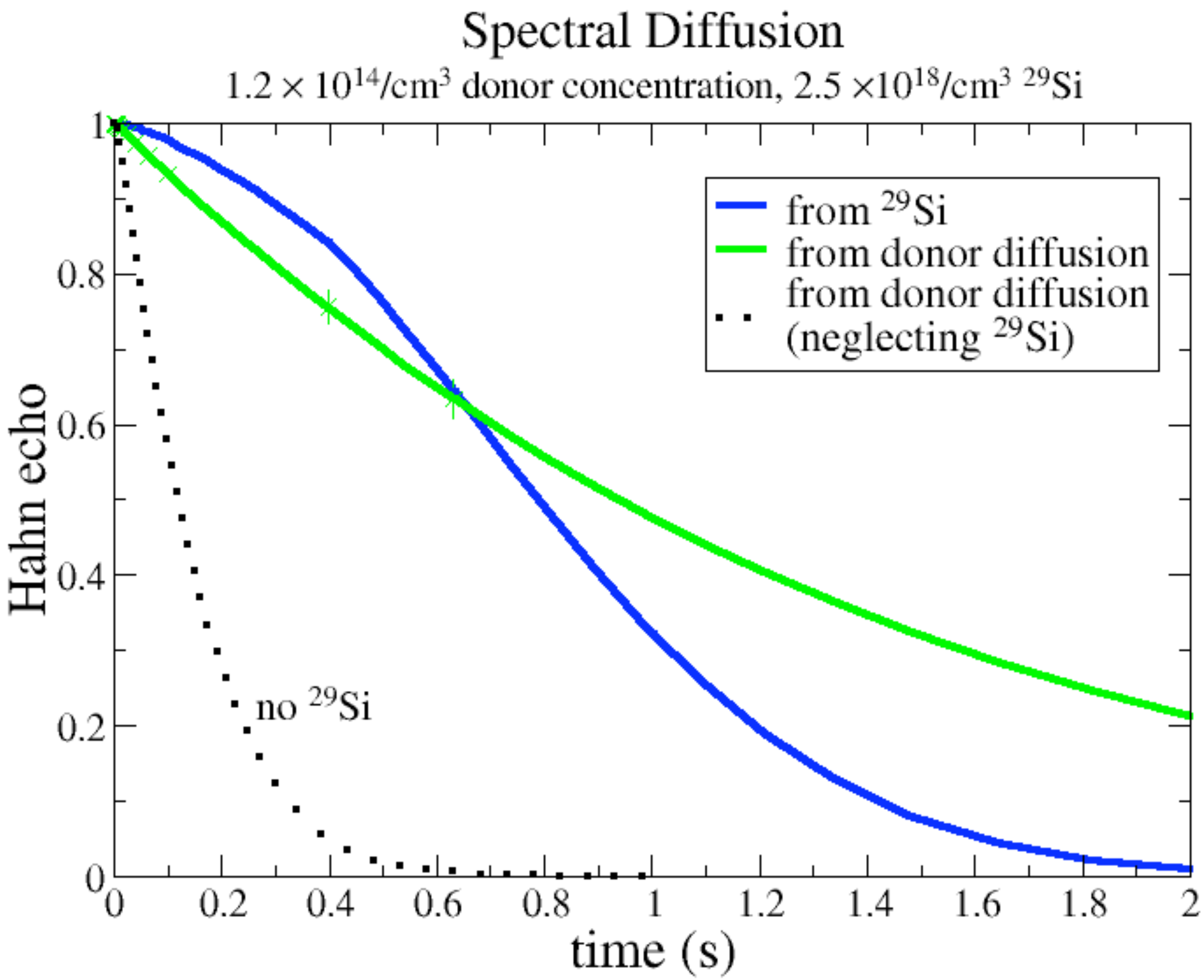


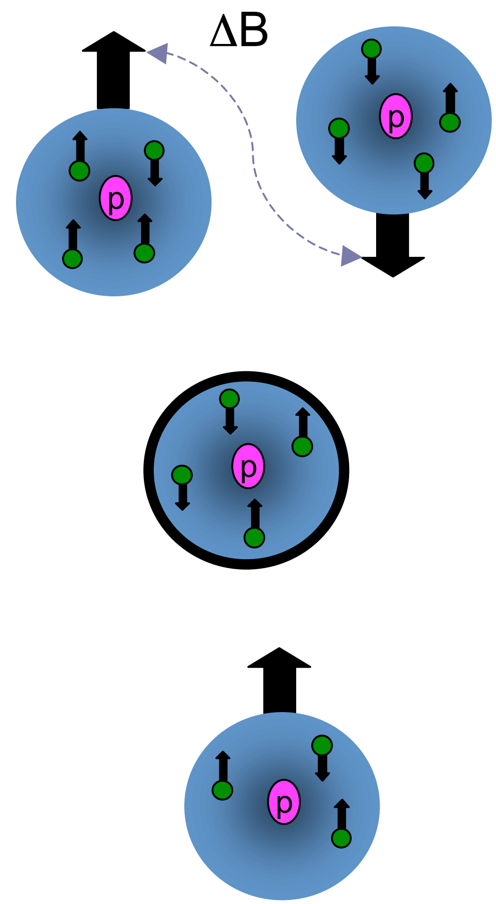
Theory: donor diffusion noise (neglecting ^{29}Si)



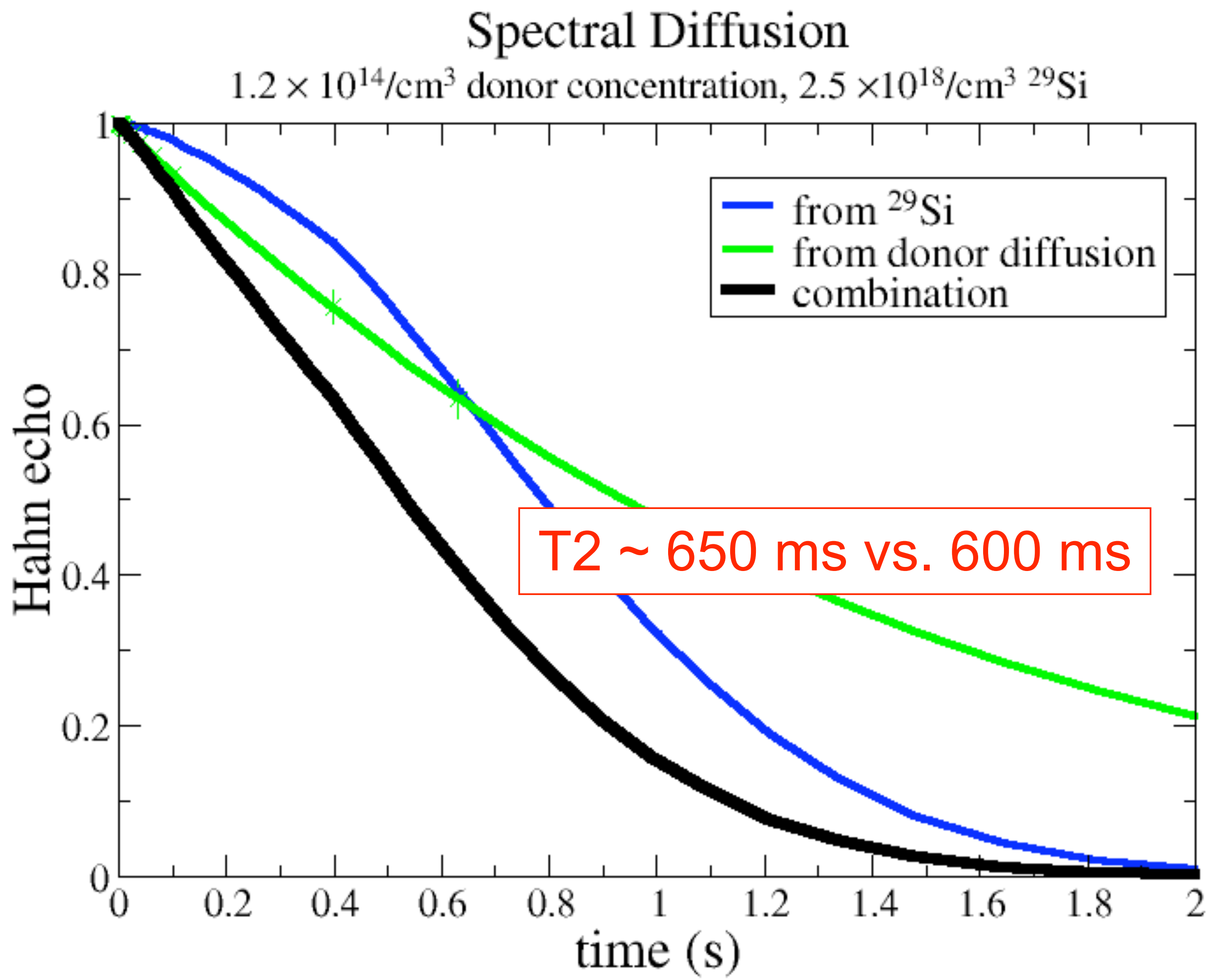


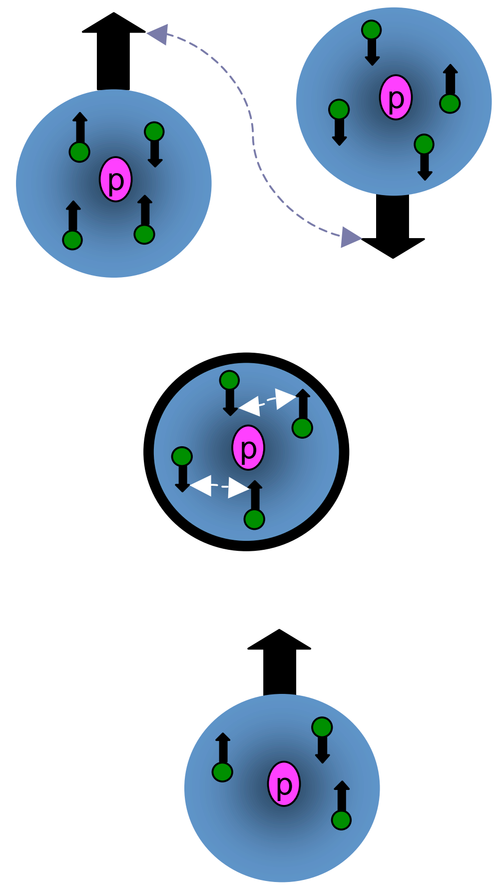
Theory: donor diffusion noise (with ^{29}Si)



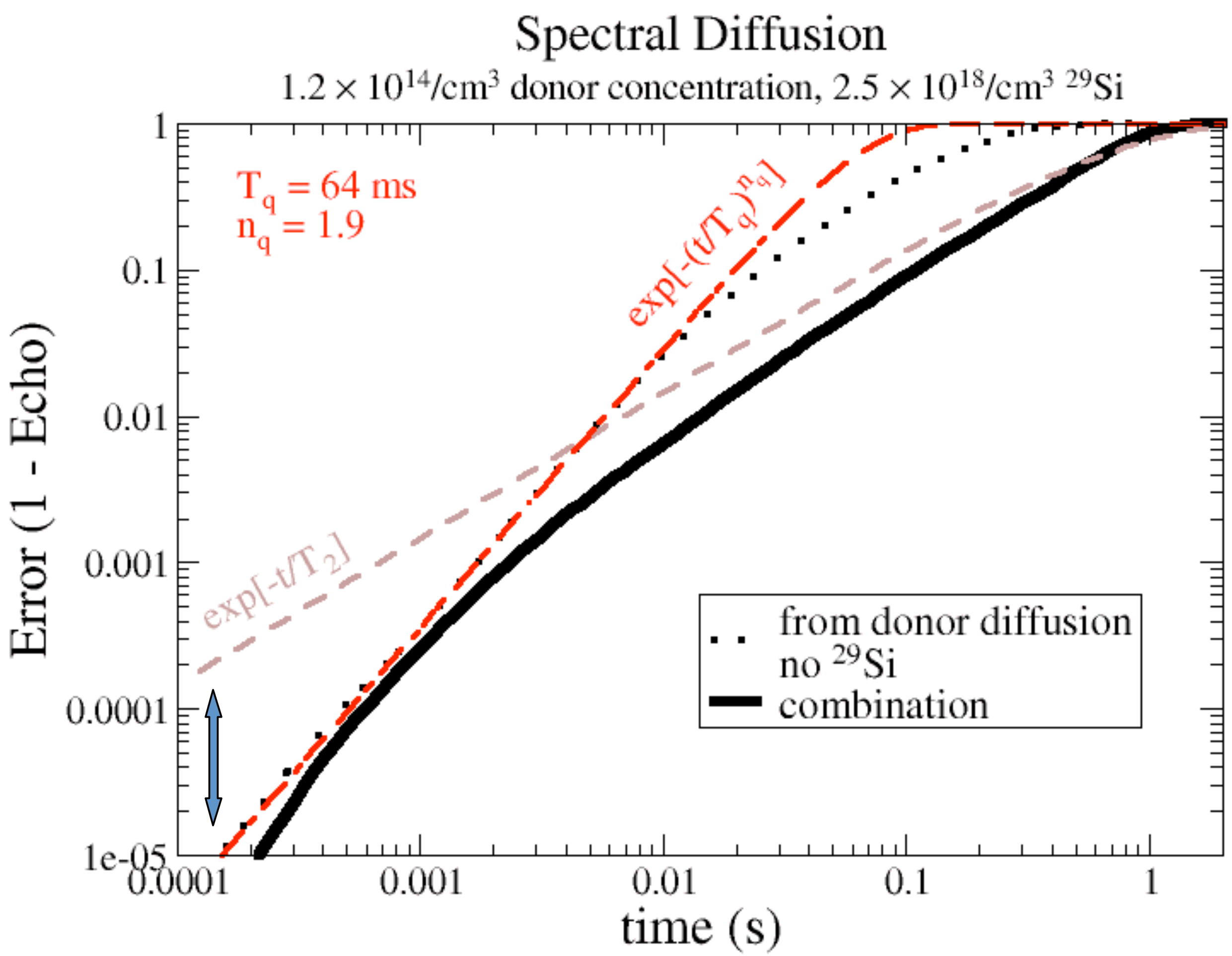


Theory: donor diffusion noise and ^{29}Si noise





Benefit of theory: low error behavior

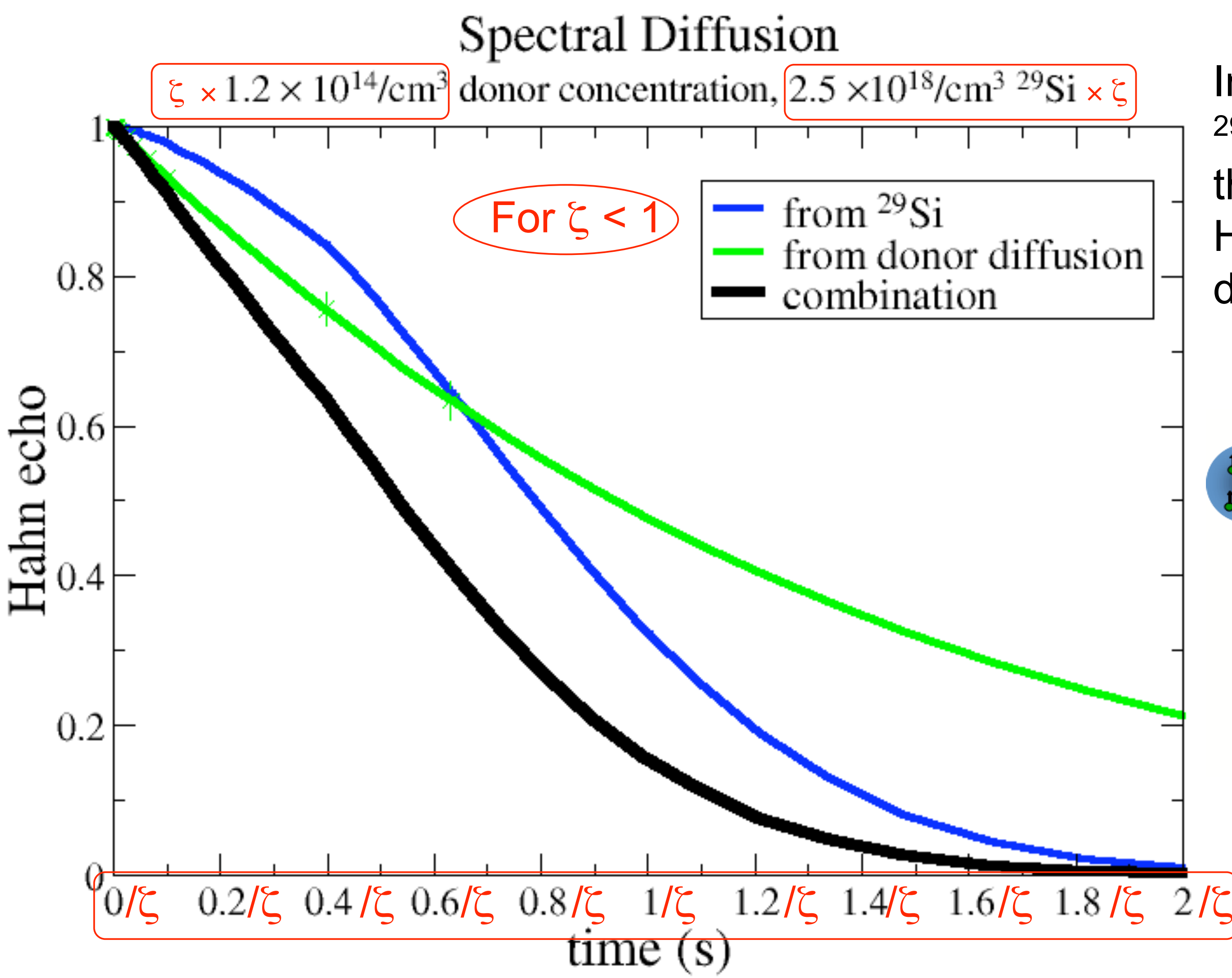


T_2 is a poor figure of merit for Q. C. where the low error regime is most important.

We introduce T_q and n_q to characterize the low error regime.

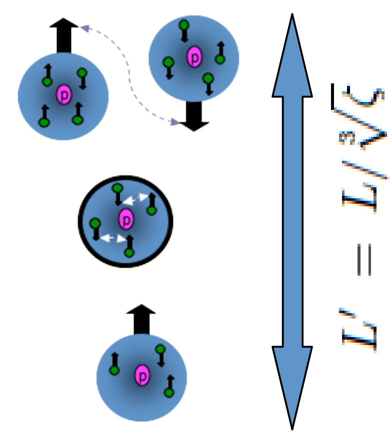
Sandia National Laboratories

Benefit of theory: changing concentration



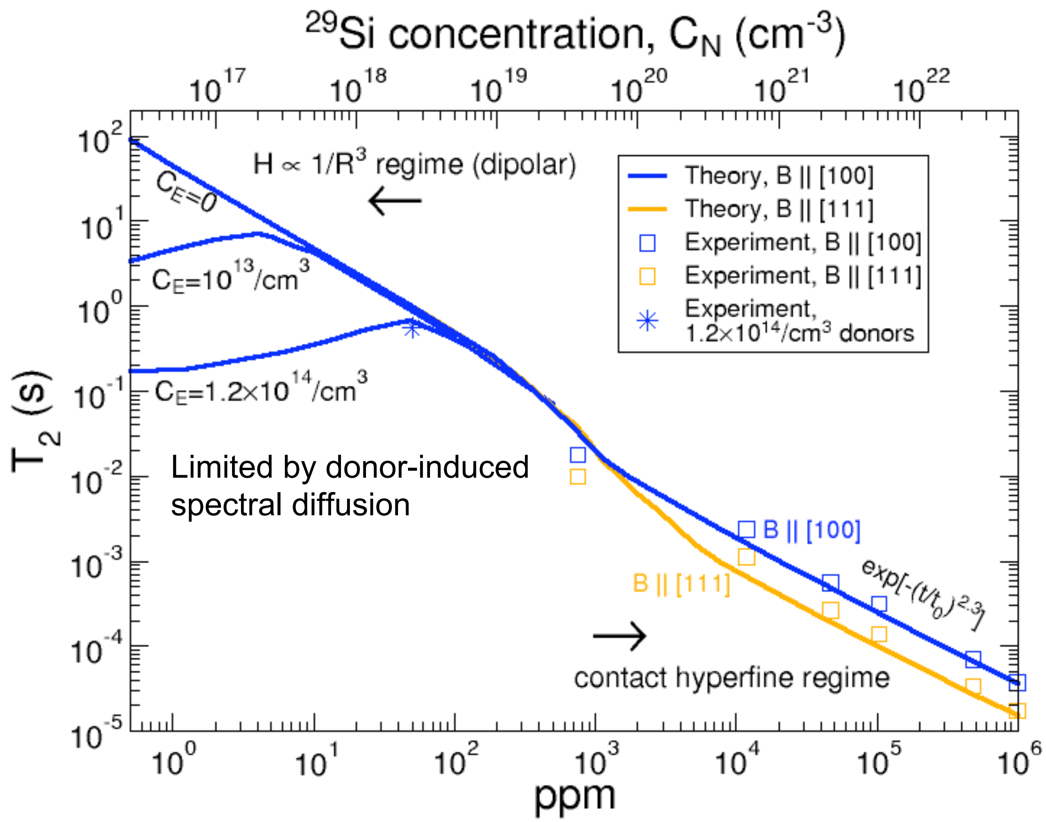
In this regime of low ^{29}Si and high B-field, the effective Hamiltonian has only dipolar interactions.

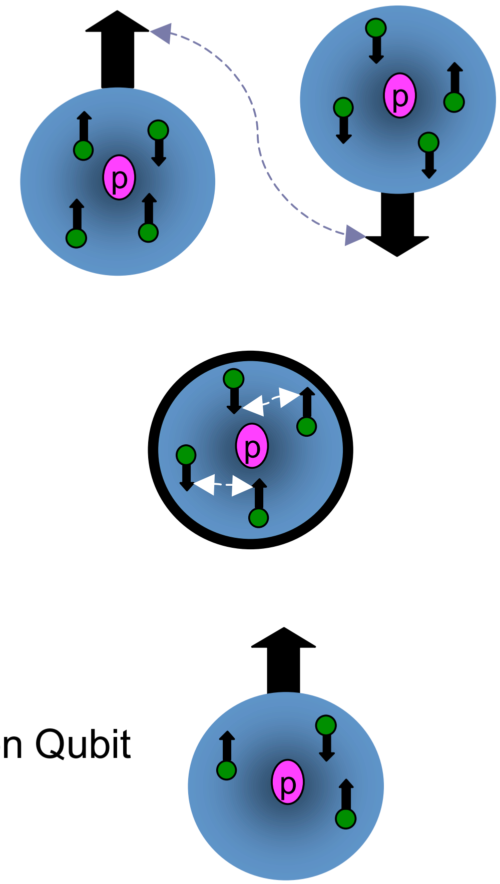
$\hat{\mathcal{H}} \propto 1/R^3$



$\hat{\mathcal{H}}' = \zeta\hat{\mathcal{H}}$ or $t' = t/\zeta$

Benefit of theory: changing concentration

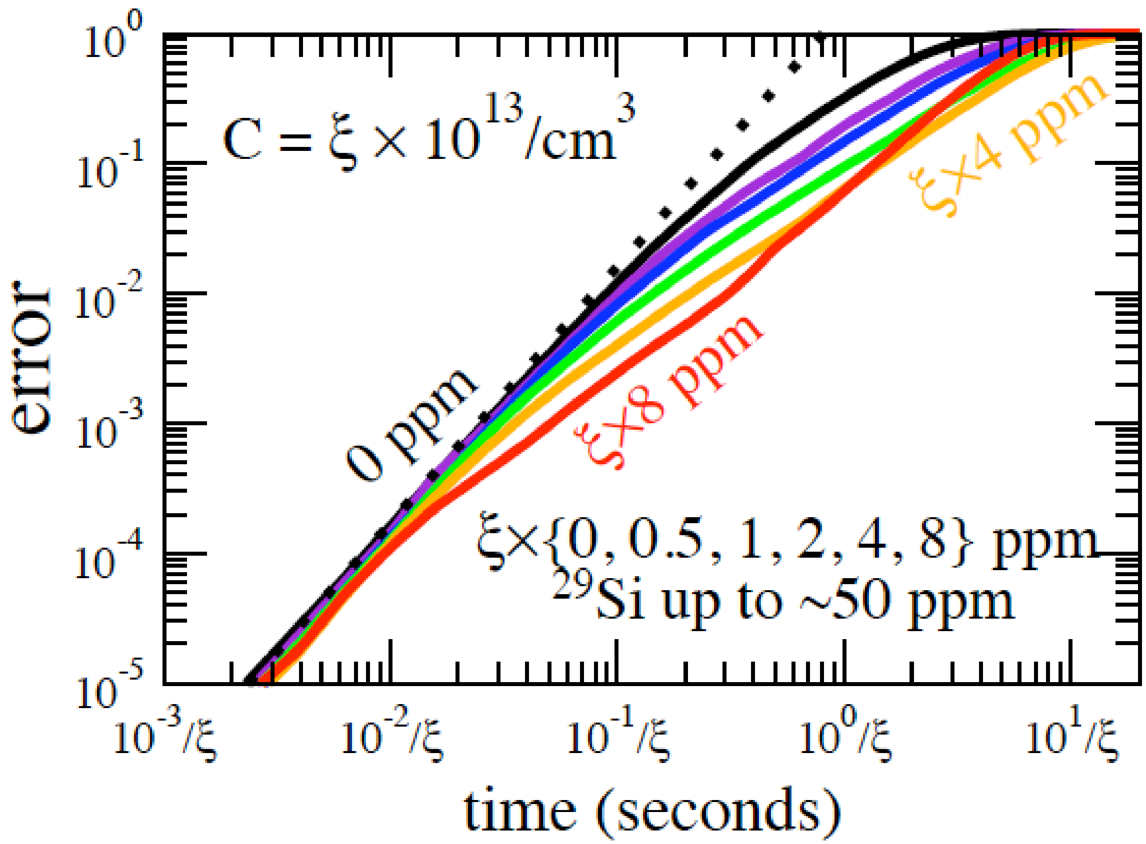


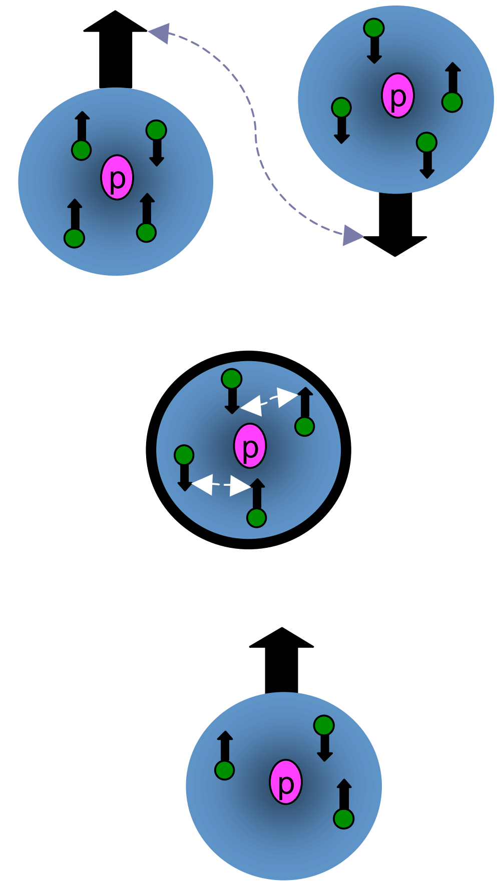


Experiments: E. Abe et al., preprint (2010); A. M. Tyryshkin et al., Silicon Qubit Workshop, August 2009, Berkeley, CA; A. M. Tyryshkin and S. A. Lyon, private communications.

What are the limits of coherence in Si as we isotopically enrich?

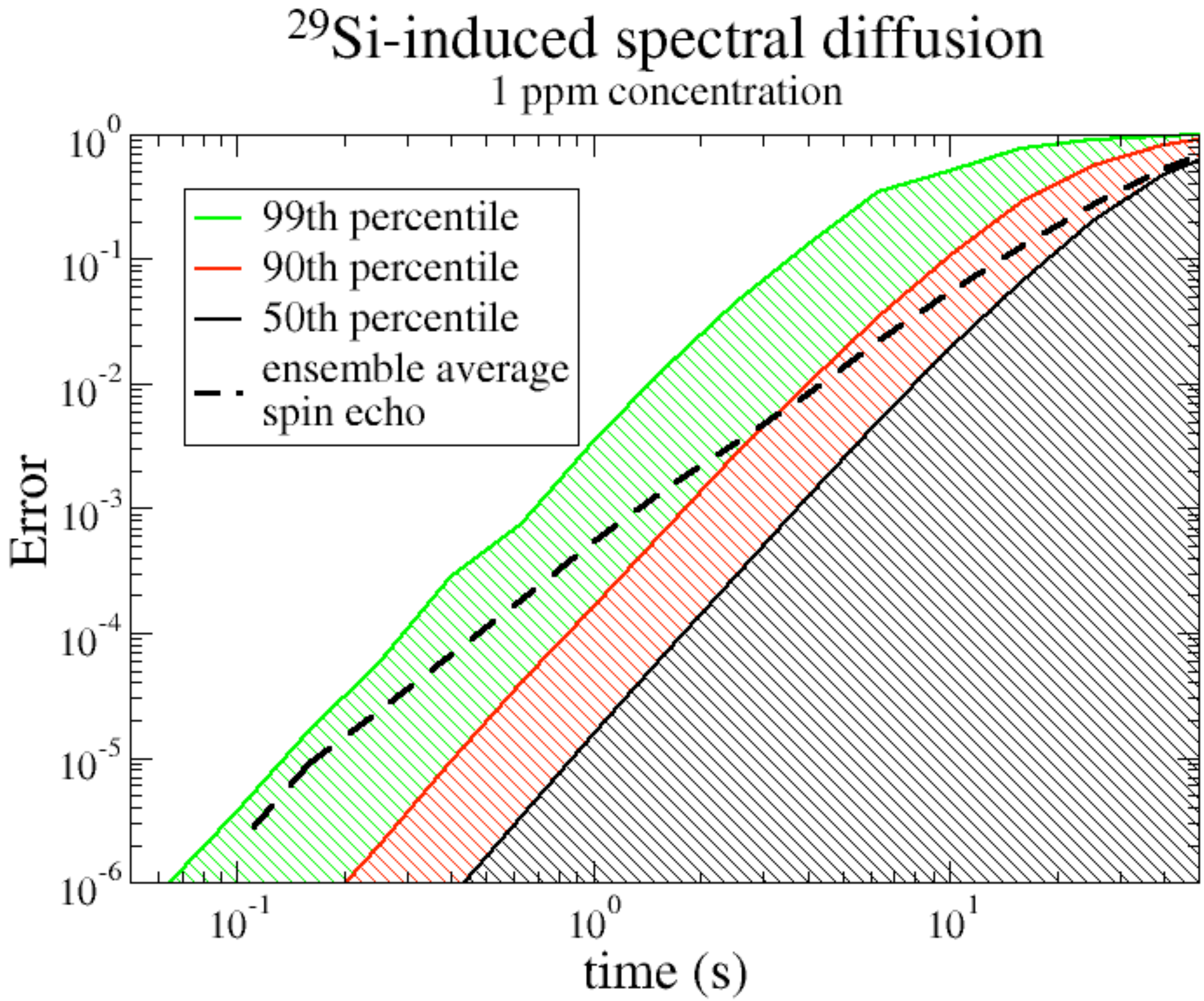
Benefit of theory: changing concentration

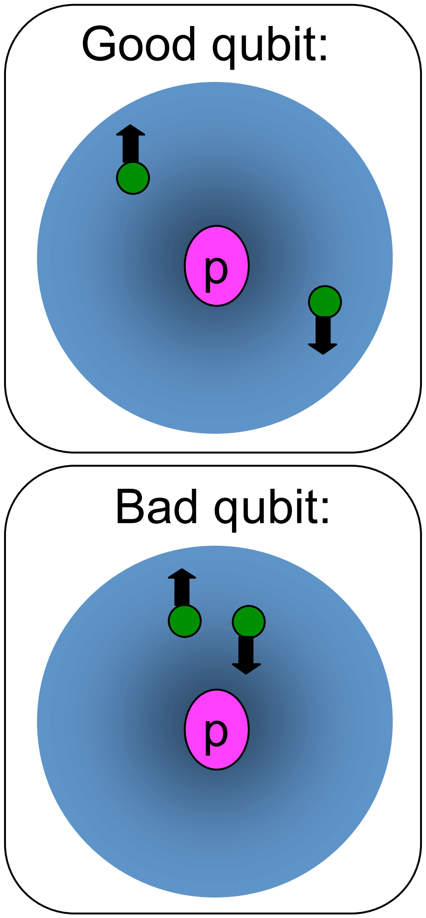




What about the short time behavior?

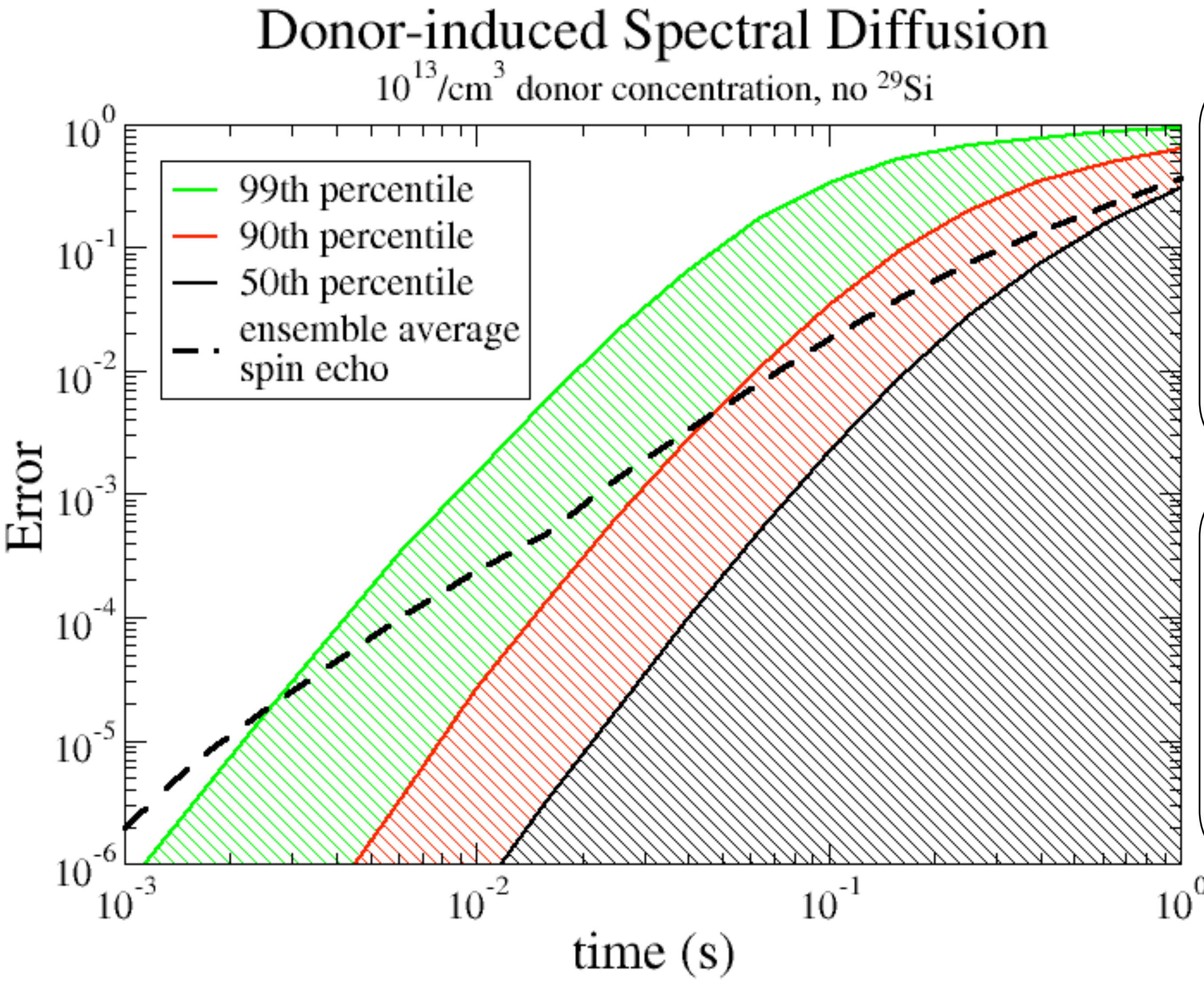
Benefit of theory: sample-to-sample variation (^{29}Si decay)

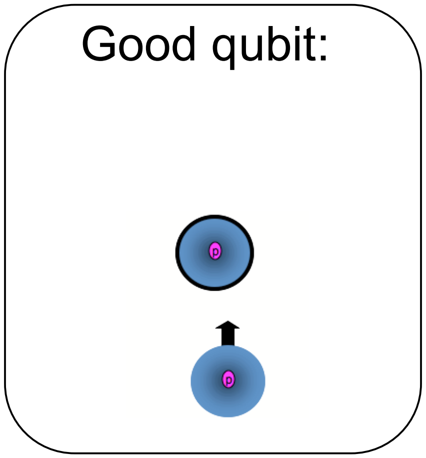


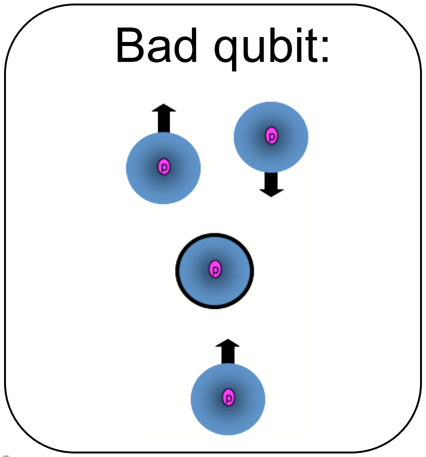


Sandia National Laboratories

Benefit of theory: sample-to-sample variation (donor decay)

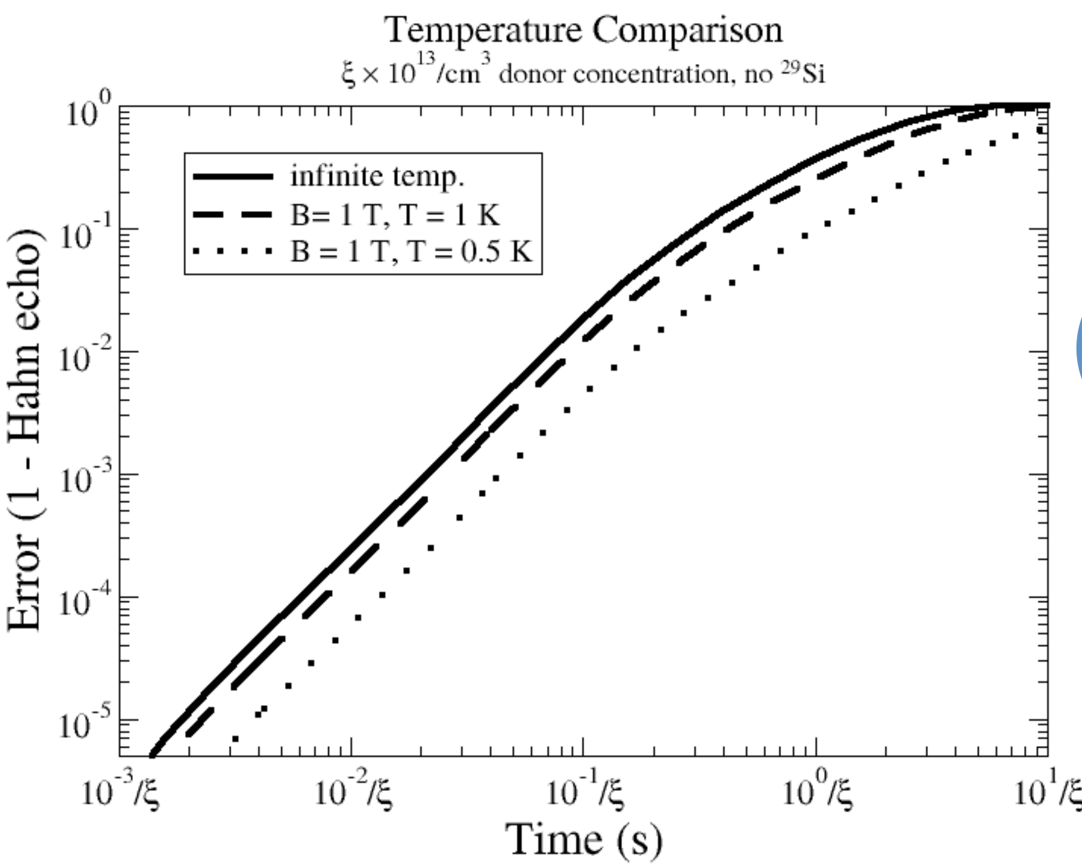


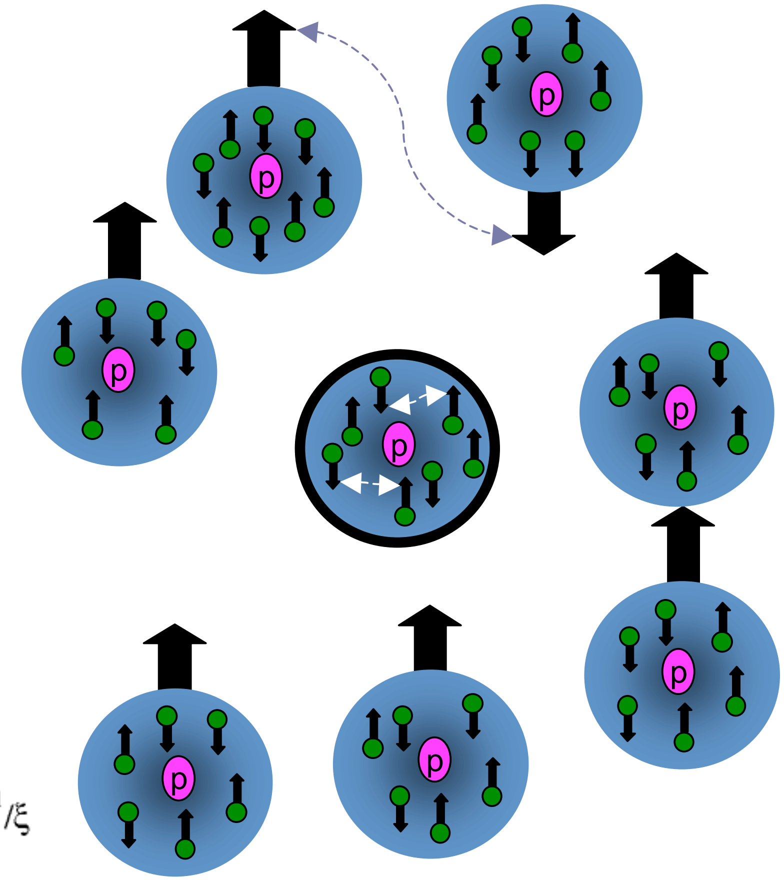




Sandia National Laboratories

Benefit of theory: low temperature





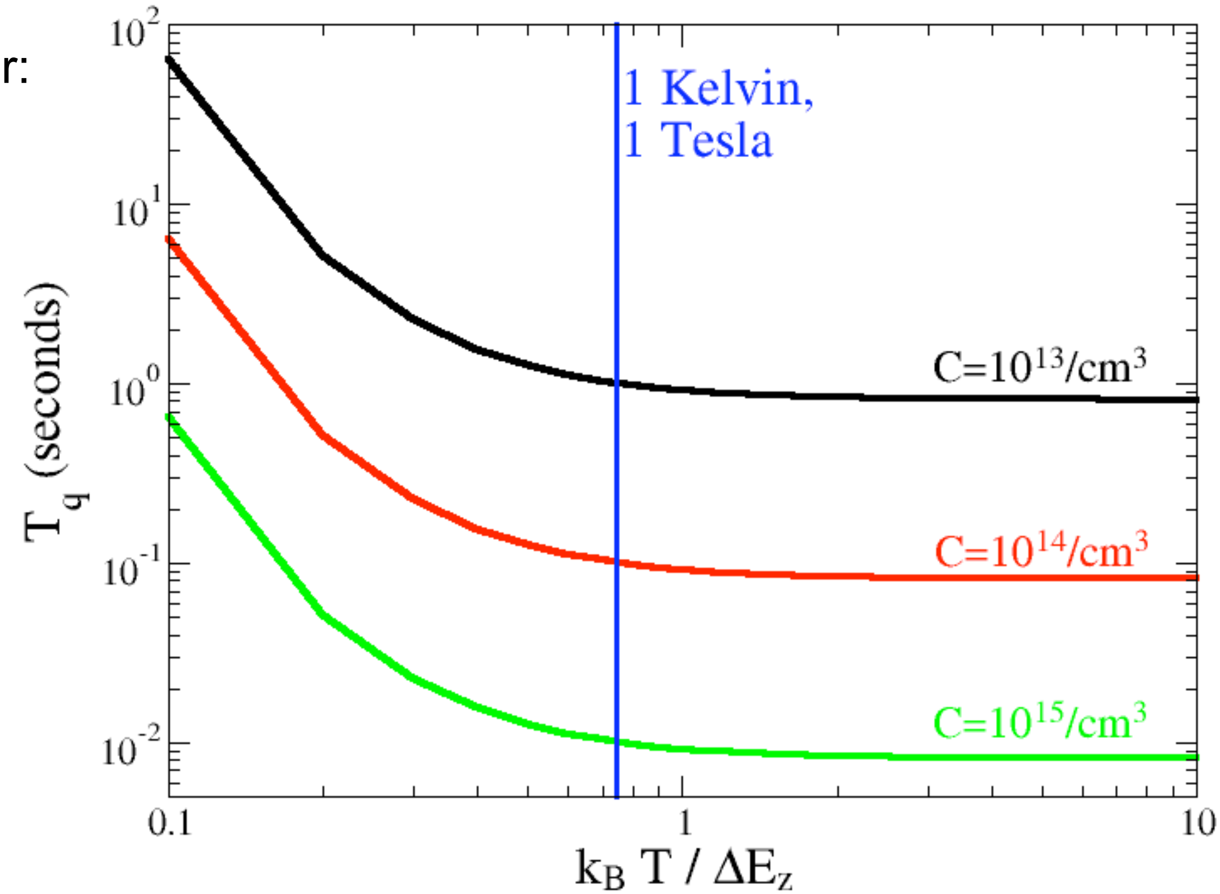
We can thermally polarize the electron spins at high enough B-field and low enough temperature.

Benefit of theory: low temperature

From considerations of spin noise only

Short time decay behavior:

$$\exp\left[-(t/T_q)^{n_q}\right]$$



In 2-cluster approx:

$$T_q \;\propto\; (p_\uparrow p_\downarrow)^{-n_q}$$

$$p_{\uparrow/\downarrow} \;=\; \exp\left(\pm E_z/k_B T\right)/2\cosh\left(E_z/k_B T\right)$$

Conclusions

- Cluster expansions successfully solve the central spin decoherence problem.
- We have adapted the Cluster Correlation Expansion technique to have the versatility to solve problems of sparse electron spin baths.
- Coherence may be limited by ^{29}Si *or* background donors.
- Having some ^{29}Si improves T_2 at low donor concentrations.
- Behavior is different in the short-time (low-error) regime.
- For low errors, both ^{29}Si and dopants should be reduced or polarized.
- There is a large sample-to-sample variation in baths of sparse spins; yield is more informative than average decoherence.

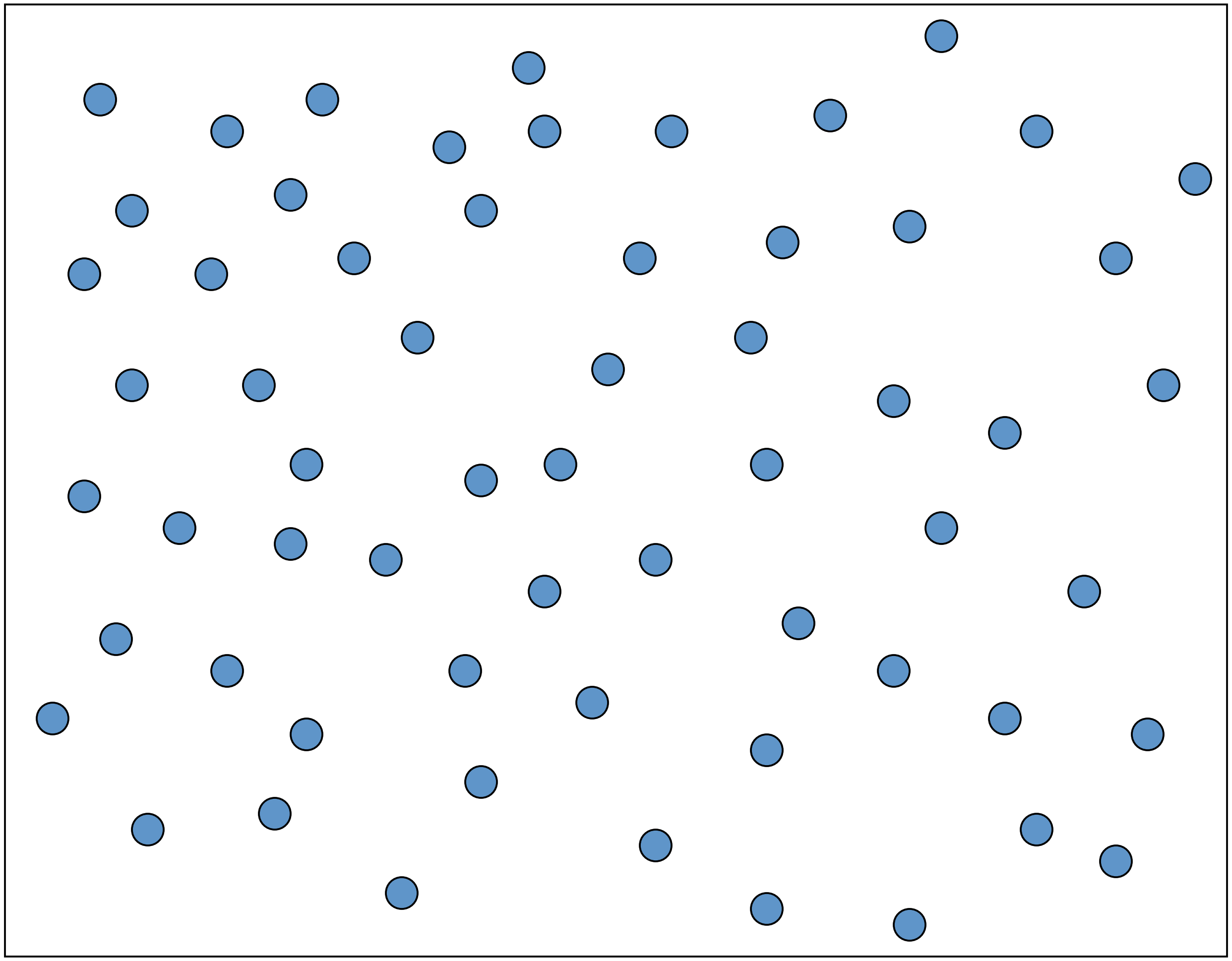
Acknowledgements and Other Applications

W. M. Witzel, M. S. Carroll, A. Morello, Lukasz Cywinski, and S. Das Sarma, arXiv:1008.2382

We thank A. Tyryshkin, S. Lyon, C. Tahan, R. Muller, E. Nielsen, R. Rahman, A. Ganti, and A. Landahl for their comments.

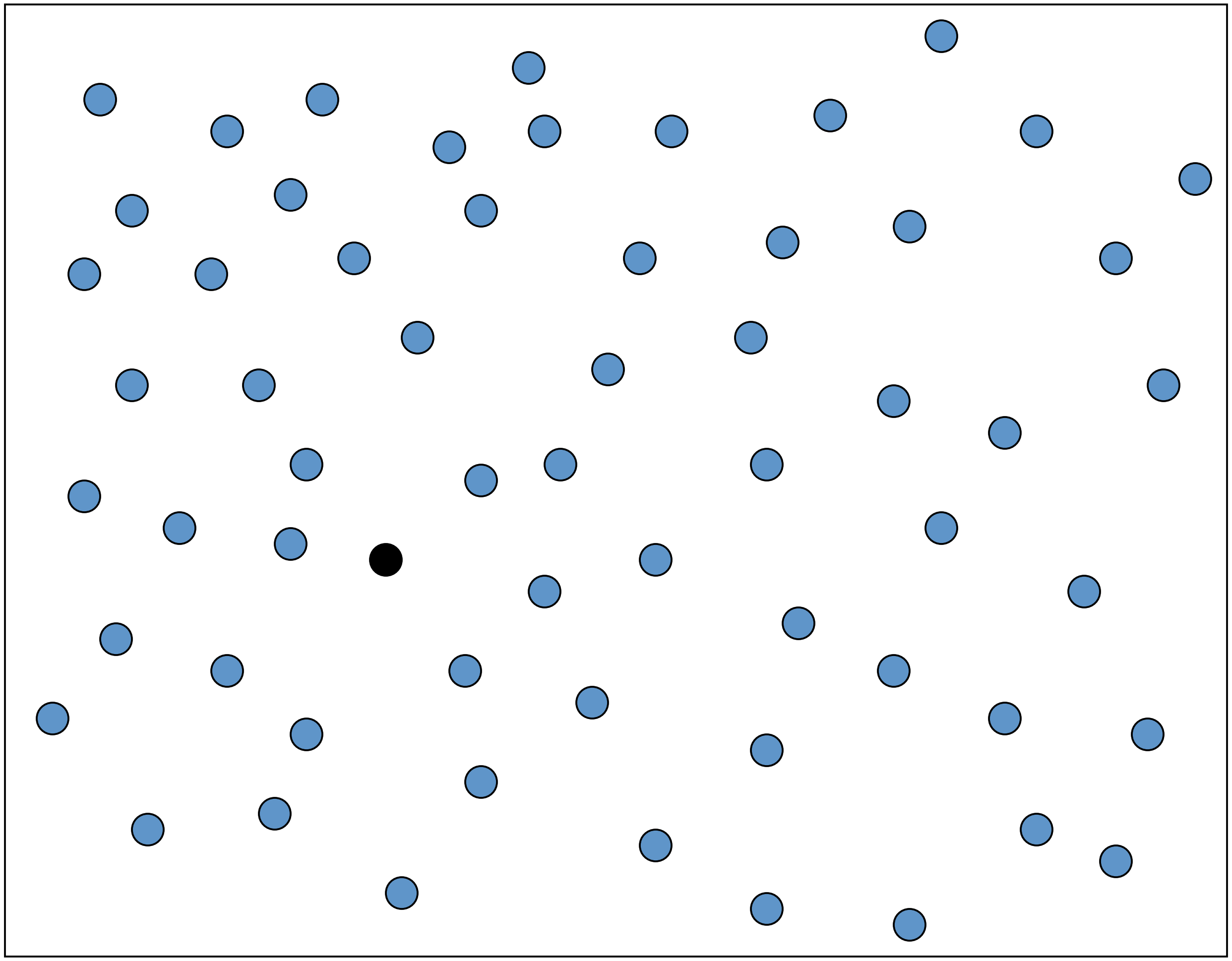
- Dangling bond spins at an interface (preparing manuscript).
- Un-ionized donors from modulation doping.
- Germanium nuclear spins.
- Any other miscellaneous spins that keep Malcolm awake at night.

Cluster Selection Heuristics



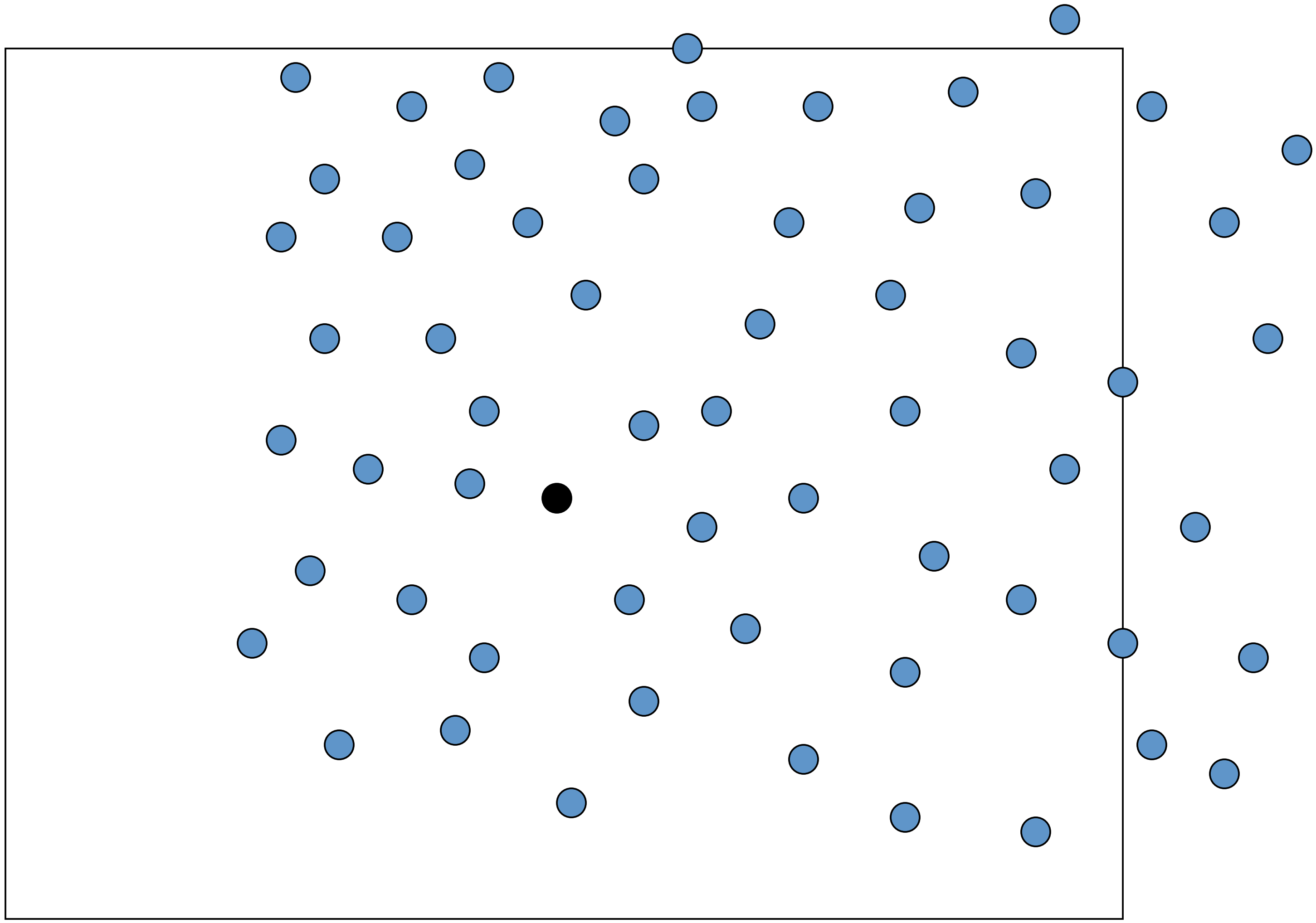
randomly generate spatial configuration of spins

Cluster Selection Heuristics



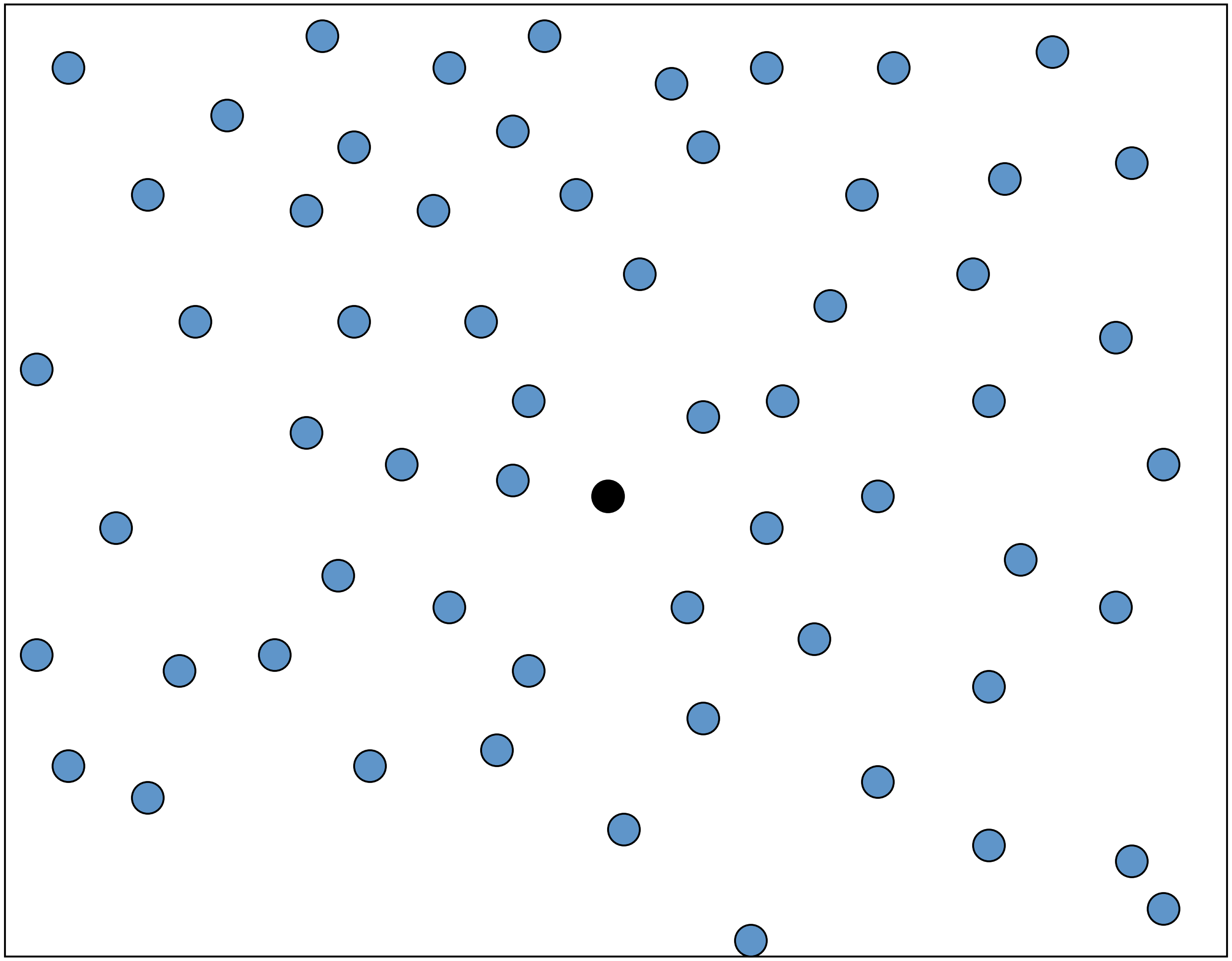
choose central spin

Cluster Selection Heuristics



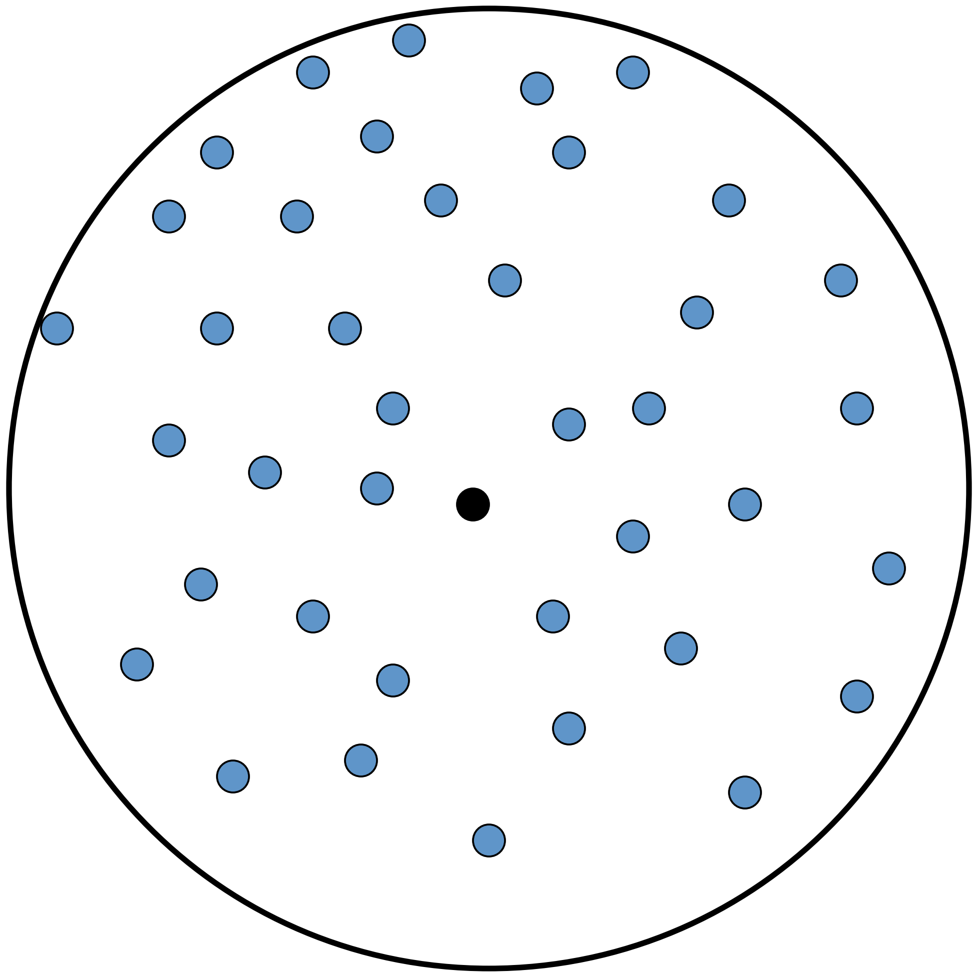
shift to center

Cluster Selection Heuristics



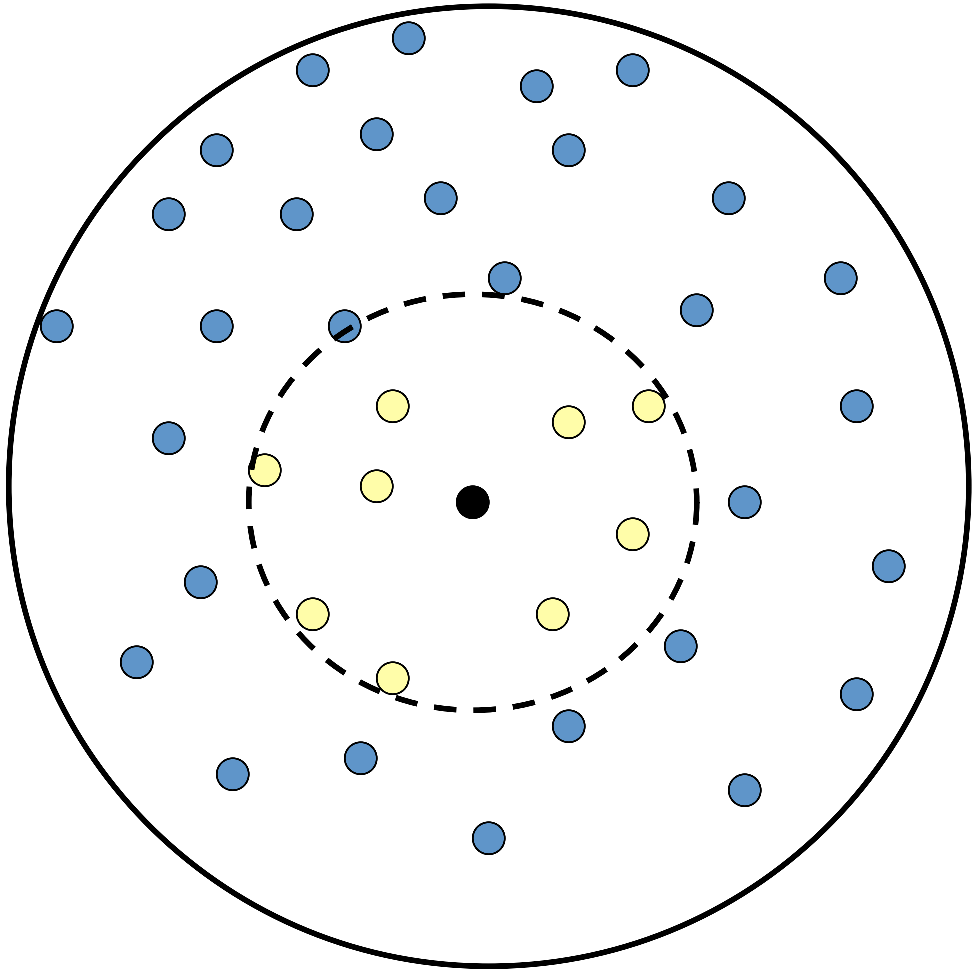
wrap around

Cluster Selection Heuristics



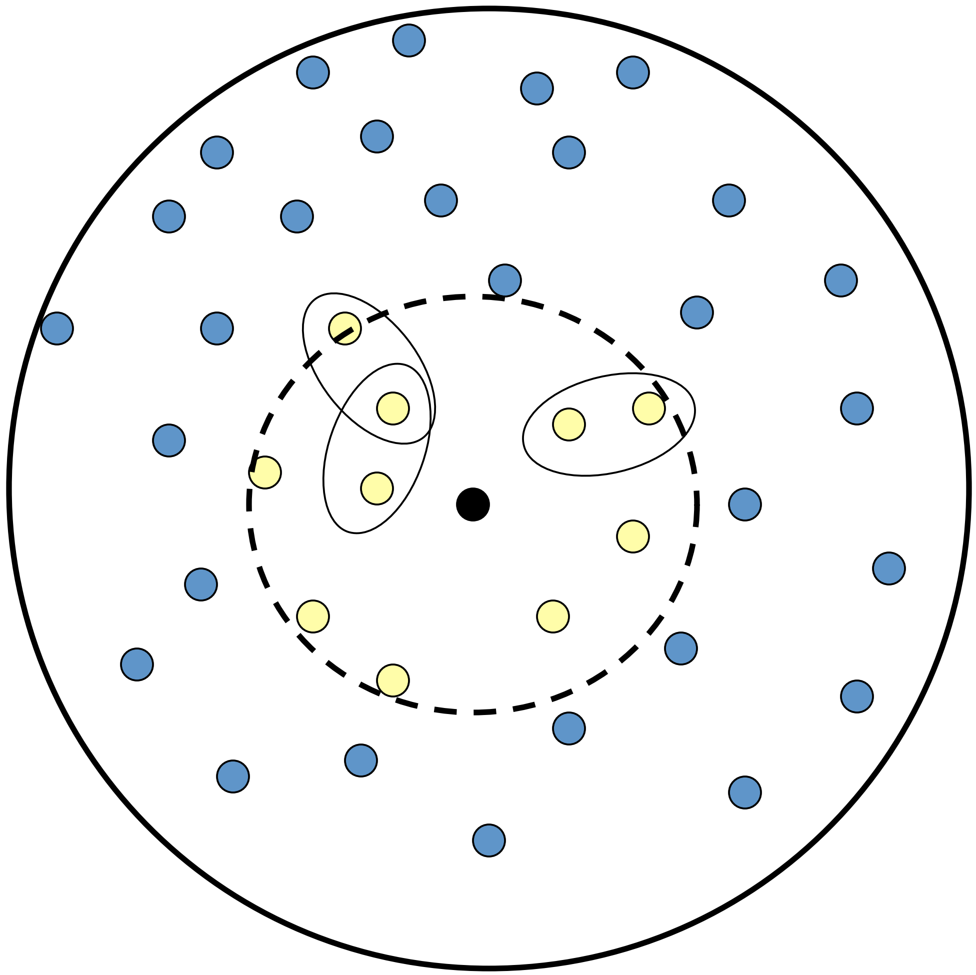
Crop out some “world radius” cutoff

Cluster Selection Heuristics



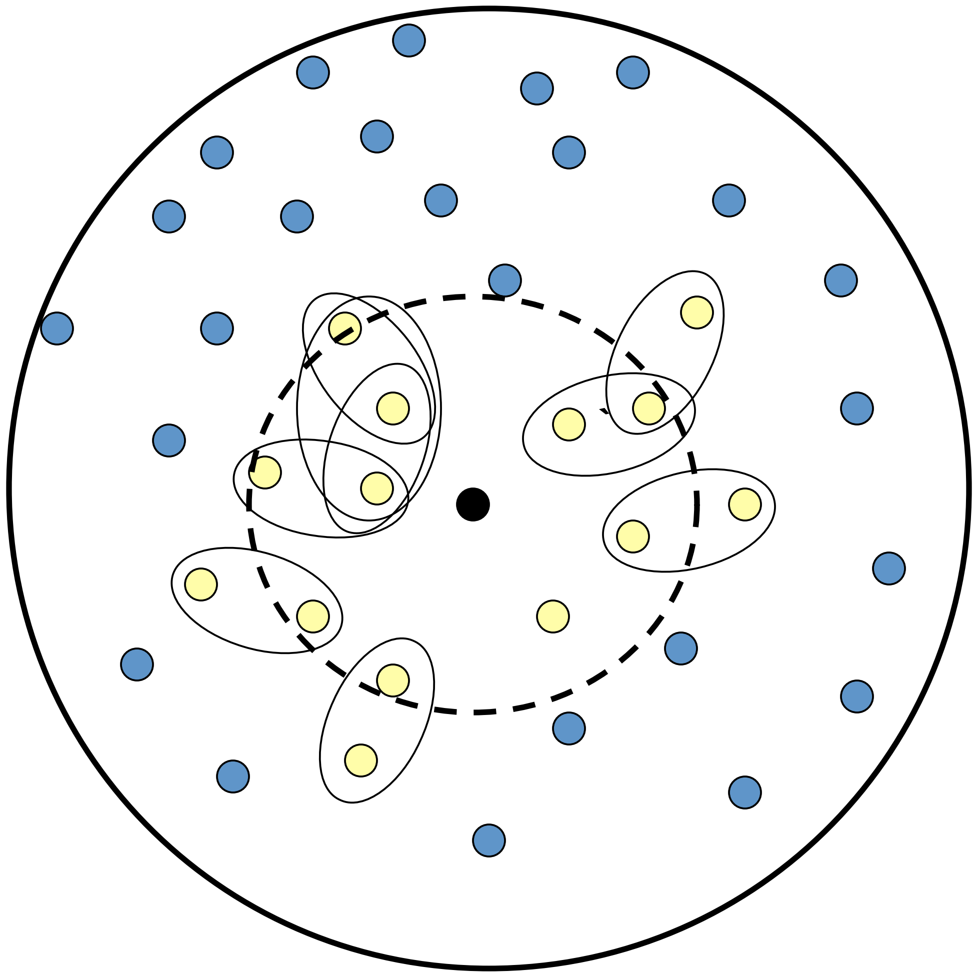
cluster “seeds” within “inner radius”

Cluster Selection Heuristics



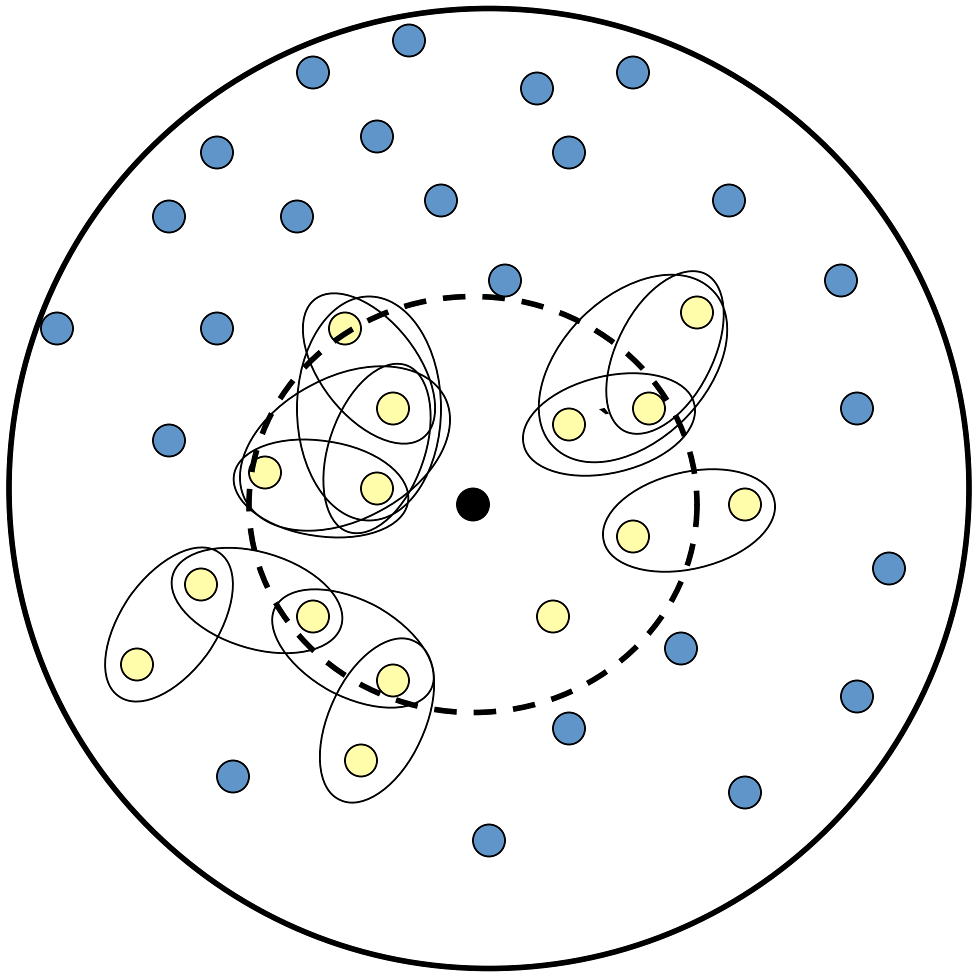
grow clusters - strongest interactions first

Cluster Selection Heuristics



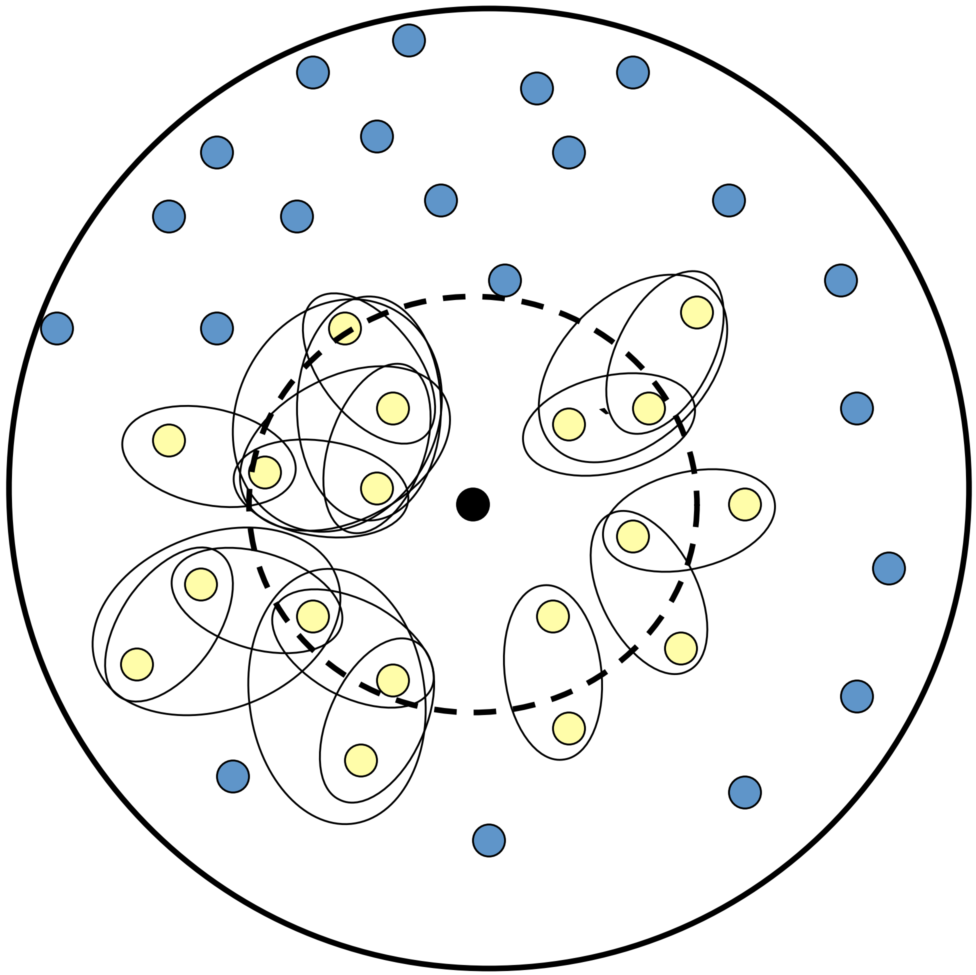
grow clusters - strongest interactions first

Cluster Selection Heuristics



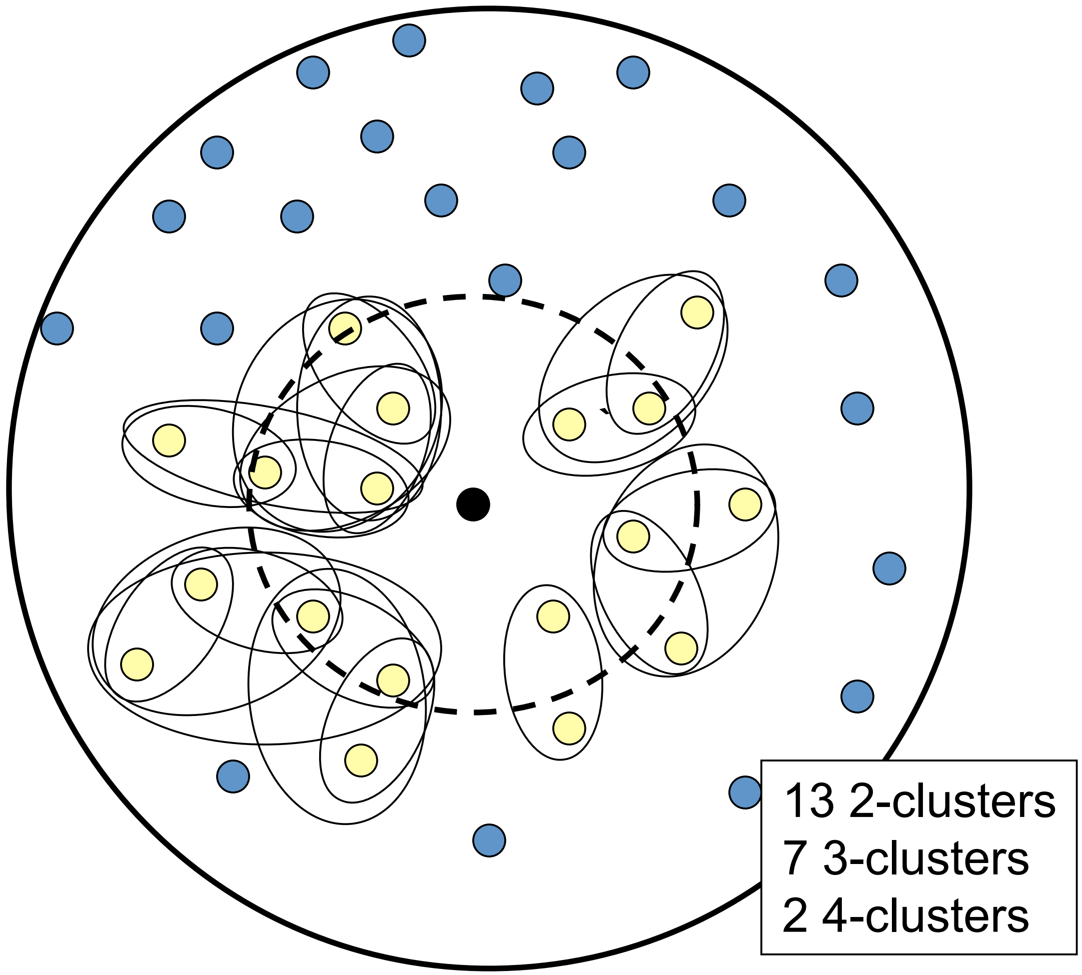
grow clusters - strongest interactions first

Cluster Selection Heuristics



grow clusters - strongest interactions first

Cluster Selection Heuristics



grown until we reach quota for each k-cluster

Sparse Electron Spin Systems Present New Challenges