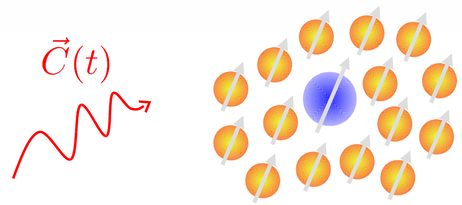
Combining DD with Optimal Control for Improved QIP

Matthew Grace[1], Wayne Witzel[1], Jason Dominy[2], and Malcolm Carroll[1]
[1]Sandia National Laboratories and [2]Princeton University

DQD Effective One-Qubit Model



$$H = H_0 - \vec{\sigma}\cdot\vec{C}(t) - \vec{\sigma}\cdot\vec{\Gamma} + H_e$$

$$\rightarrow H = \sigma_z C_z(t) + \epsilon\sigma_x$$

Different magnetic fields at each dot produce a rotation about the logical x-axis

Pulse area: $\theta(t) = \int_0^t \vec{C}(\tau)\mathrm{d}\tau$

Dynamical-Decoupling Sequences

$\prod_i^N U_i \approx \mathbb{1}$, where U_i represent π and $\pi/2$ rotations and free evolutions.

This is an approximation to $\mathbb{1}$ because

- $\{U_i\}$ and N are finite
- Non-unitary evolution is corrected with unitary "time-reversal" operations

Dynamical-Decoupling Pulses

To remove 1st- and 2nd-order errors in π and $\pi/2$ z-axis rotations when $\vec{\Gamma} = \Gamma_x \Rightarrow \vec{\eta} = 0$:

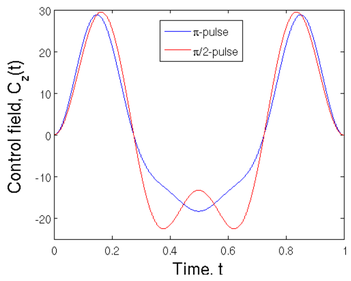
$$\eta_1 = \int_0^{t_f} \sin[\theta(t)]\mathrm{d}t, \quad \eta_2 = \int_0^{t_f} \cos[\theta(t)]\mathrm{d}t,$$

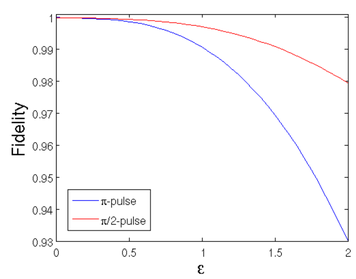
$$\eta_3 = \int_0^{t_f} t\sin[\theta(t)]\mathrm{d}t, \quad \eta_4 = \int_0^{t_f} t\cos[\theta(t)]\mathrm{d}t,$$

$$\eta_5 = \int_0^{t_f}\int_0^{t_f} \sin[\theta(t_1) - \theta(t_2)]\mathrm{sign}(t_1 - t_2)\mathrm{d}t_1\mathrm{d}t_2,$$

S. Pasini, *et al.*, Phys. Rev. A, **80** (2009)

Feasible controls satisfying $\vec{\eta} = 0$





Resulting Fidelities: $\frac{1}{2}\left|\mathrm{Tr}\left[U(t_f)Z_\theta^\dagger\right]\right|$

Optimal Dynamical-Decoupling Pulses

By searching the space of controls satisfying

1. $\vec{\eta} \approx 0$
2. $\mathcal{F} = \frac{1}{2}\left|\mathrm{Tr}\left[U(t_f)Z_\pi^\dagger\right]\right| \approx 1$

we improve control fidelities and system robustness for $Z_{\pi/2}$ and Z_π.

Systematic searching $\Rightarrow$ Optimal control theory

Optimal Control Theory for Quantum-Mechanical Objectives

•Define an objective: $\mathcal{J}\left[\vec{C}(t)\right] = \frac{1}{2}\left|\mathrm{Tr}\left[U(t_f)Z_\theta^\dagger\right]\right|$

•Incorporate constraints:

- Schrödinger's equation
- Experimental limitations of the control field

$$\mathcal{J}' = \mathcal{J} + \mathcal{S}[U(t)] - \alpha\int_0^{t_f}\left|\vec{C}(t)\right|^2\mathrm{d}t$$

•Perform variational analysis and optimize iteratively

- Evolutionary algorithms
- Gradient-based methods

$$\frac{\delta\mathcal{J}'[\vec{C}(t)]}{\delta\vec{C}(t')} = 0$$

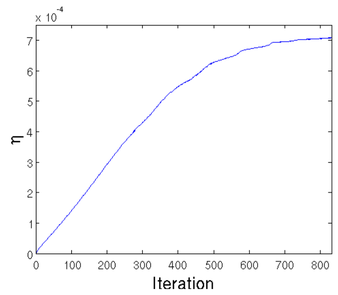
Level Sets and Tangent Spaces

After calculating $\frac{\delta\mathcal{J}'}{\delta\vec{C}(t)}$, all gradient directions $\frac{\delta\eta_i}{\delta\vec{C}(t)}$ are removed:

$$\frac{\delta\mathcal{J}'}{\delta\vec{C}(t)} \longrightarrow \frac{\delta\mathcal{J}'}{\delta\vec{C}(t)} - \sum_i p_i\frac{\delta\eta_i}{\delta\vec{C}(t)}$$

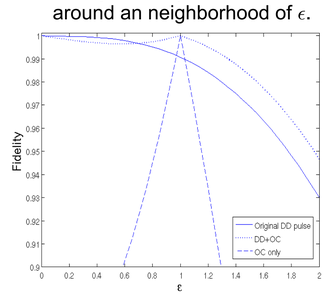
so $\left\langle\frac{\delta\mathcal{J}'}{\delta\vec{C}(t)}, \frac{\delta\eta_i}{\delta\vec{C}(t)}\right\rangle = 0.$

Optimal Dynamical-Decoupling Results: Z_π



$\|\vec{\eta}\|$ $10^{-8} \rightarrow 10^{-4}$

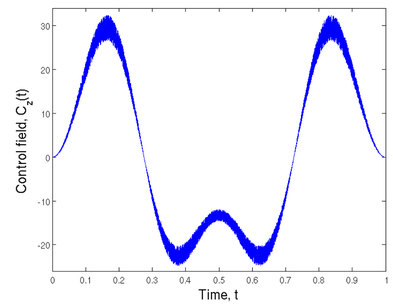
DD+OC yields improved robustness around an neighborhood of ϵ.



$\mathcal{F}$ $0.99 \rightarrow 0.9999$ at $\epsilon = 1.0$

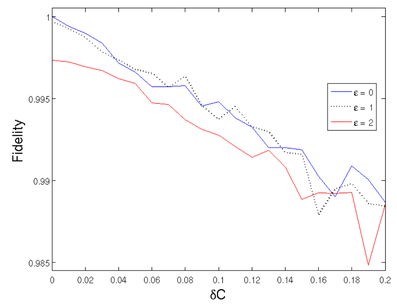
Fidelity Response to Control Noise: $Z_{\pi/2}$

$C(t) \rightarrow C(t) + \delta C(t)$, where $\delta C \in \mathcal{U}$, $|\delta C(t)| = aC(t)$, and $0 \le a \le 0.2$

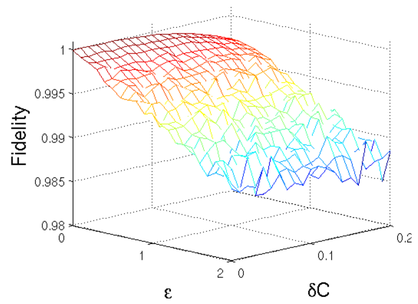


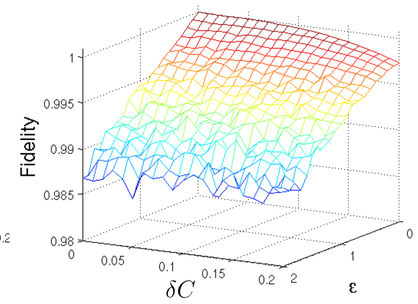
$a = 0.1$

⇒ 10% control noise



Fidelity variation dominated by ϵ





Fidelity variation dominated by ϵ; δC is significant for small ϵ.

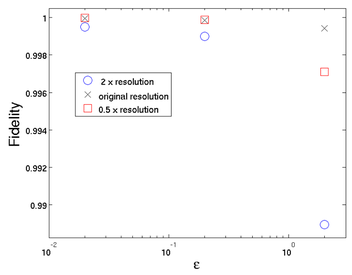
Control with Experimental Constraints

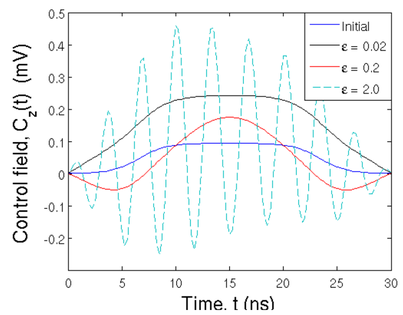
Resolution at q-dot:

•Max. voltage = ~3mV

•Step size = ~3μV

•Step time = ~1 ns





ϵ	0.02	0.2	2.0
Fidelity	0.9995	0.9999	0.9999

Current Work and Future Directions

•Incorporate these optimal π- and $\pi/2$-pulses for memory and information processing.

- •Investigate various noise models (e.g., spin baths).
- •Determine how these pulses extend spin-echoes.

•Consider ϵ as time-dependent during the application of the control field.

•Extend this formalism to arbitrary rotation axes.

•Analyze the control landscape within the region of feasibility, i.e., $\eta = 0$.

Sandia National Laboratories
This work was supported by the Laboratory Directed Research and Development program at Sandia National Laboratories. Sandia is a multi-program laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000. SAND2010-8736C