

# Congestion and Cascades in Coupled Payment Systems

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By

Fabien Renault\*, Walter E. Beyeler♦, Robert J. Glass♦,  
Kimmo Soramäki\* and Morten L. Bech•▲

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*Abstract:*

*This paper analyses liquidity and credit risks in the context of interdependent interbank payment systems. A simple model is developed to investigate the operation of two real time gross settlement systems interlinked through FX transactions conducted by a set of global banks that participate in both systems. In addition, further interdependence is created by imposing a Payment versus Payment (PvP) constraint. The model illustrates under which conditions settlement of payments in the two systems becomes correlated and how large credit exposures can be generated as the result of liquidity pressures in one of the two systems. PvP can eliminate this credit risk but will make each system dependent on the level of liquidity available in the other system.*

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\* Banque de France, [fabien.renault@banque-france.fr](mailto:fabien.renault@banque-france.fr).

♦ Sandia National Laboratories, [webeyel@sandia.gov](mailto:webeyel@sandia.gov) and [rjglass@sandia.gov](mailto:rjglass@sandia.gov). Messrs. Beyeler and Glass acknowledge the financial support of the National Infrastructure Simulation and Analysis Center (NISAC), a program under the Department of Homeland Security's (DHS) Preparedness Directorate. Sandia National Laboratories (SNL) and Los Alamos National Laboratory (LANL) are the prime contractors for NISAC under the programmatic direction of DHS's Infrastructure Protection/Risk Management Division. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

• European Central Bank and Helsinki University of Technology, [kimmo@soramaki.net](mailto:kimmo@soramaki.net).

• Federal Reserve Bank of New York, [morten.bech@ny.frb.org](mailto:morten.bech@ny.frb.org) (corresponding author).

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## **Executive Summary**

This paper presents a simple model that describes the operation of two RTGS payment systems, operating in two distinct currencies, and interacting with each other through FX transactions performed by a set of global banks that participate to both systems. This dual participation, and the resulting common inflow of FX trades, creates an interlinkage between the two systems. In addition, an additional constraint can be put on the system by imposing that the FX transactions are settled PvP.

The model was able to capture how, due to those two interdependencies, the two systems can become correlated, in the sense that a period of high settlement rate in one system will statistically correspond to a period of high settlement rate in the other system.

When the FX trades are settled non-PvP, some credit exposures are created between the global banks that engage in FX trading. Those exposures are shown to be dependent on the level of liquidity present in each system. Moreover, it appears that a structural liquidity imbalance between the two systems leads to very high exposures, by acting in a similar way as a time zone difference between the two systems.

In the PvP case, the results show that the average level of queuing within one RTGS does not depend only on its own level of liquidity like in the non-PvP case, but also on the level of liquidity in the other system. More specifically, when liquidity is decreased within the “less liquid” system, the level of queuing increases significantly within the “more liquid” system. In addition, we also observe that the level of queuing in the “less liquid” system decreases when the liquidity is increased in the “more liquid” one.

The proposed approach could be of interest to Central Banks, as a growing attention is now being given to the question of system interdependencies. In this context, the presented model can already provide a qualitative description of the consequences of the interdependency created by FX transactions on the activity of two systems.

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# 1. Introduction

Central Banks are currently noticing a tendency towards a greater interdependence between the world's payment and settlement systems. This phenomenon has multiple causes. First, consolidation in the banking sector is creating large multinational institutions that participate in several different systems. Hence, some systems are becoming interlinked through a set of common participants or "global players". Another reason behind the strengthening of the system interdependencies lies in the development of mechanisms designed to ensure *delivery-versus-payment (DvP)* in securities settlements or *payment-versus-payment (PvP)* in FX trades. While those mechanisms ensure the system participants bear no credit risk, they also make the smooth functioning of one system dependent on another system's liquidity and continued operation.

Given the importance of payment and settlement systems with regard to financial stability, Central Banks need to understand and assess the potential consequences of such an evolution. Indeed, in 2001, the Group of Ten "Report on Consolidation in the Financial Sector" (the Ferguson report) reported that "the emergence of multinational institutions and specialized service providers with involvement in several payment and securities settlement systems in different countries, as well as the increasing liquidity interdependence of different systems, further serve to accentuate the potential role of payment and settlement systems in the transmission of contagion effects.<sup>1</sup>".

To complement this previous work, the Committee on Payment and Settlement Systems<sup>2</sup> (CPSS) mandated a working group to describe the different interdependencies existing among the payment and settlement systems of CPSS countries and analyze the risk implications of the different interdependencies. The CPSS Working Group on System Interdependencies conducted a fact-finding exercise to dress an accurate picture of the situation. The Group also performed some detailed case studies, to analyze how operational or financial disruptions affecting key systems, institutions, or service providers could be transmitted between two or more payment and settlement systems.

In parallel of the working group's activities, some Central Banks and research institutions investigated the issue of system interdependencies from a modeling point of view. A joint effort of the Federal Reserve Bank of New York, Sandia National Laboratories, the Helsinki University of Technology and Banque de France led to the creation of a simple simulations framework for analyzing interdependencies between RTGS systems.

This paper presents the model and the first obtained results. It is structured as follows. Section 2 presents some prior research and sets out the objectives of the current model. Section 3 provides a description of the model and its parameters. The first set of results concerning correlated behavior of the two systems is presented in section 4. The results on settlement risk in the case of non-PvP settlement are presented in section 5. Section 6 analyses the impact of adding the PvP constraint on the level of queuing in both systems. Section 7 concludes and summarizes the paper.

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<sup>1</sup> Groupe of Ten Report on Consolidation in the Financial System, January 2001 p 29, [www.bis.org](http://www.bis.org)

<sup>2</sup> The Committee on Payment and Settlement Systems (CPSS), based at the Bank for International Settlements, contributes to strengthening the financial market infrastructure through promoting sound and efficient payment and settlement systems.

## 2. Modeling system interdependencies

### 2.1. Previous research

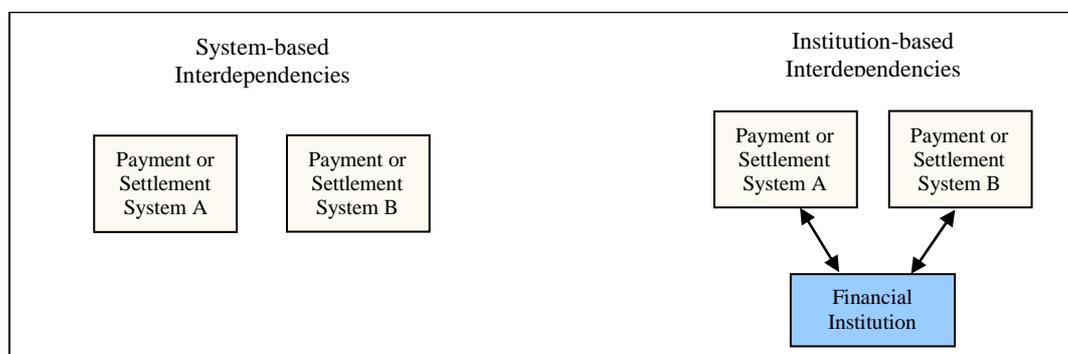
The recent development of simulation tools able to reproduce the operation of payment systems using real payment data have allowed several Central Banks to conduct stress-testing studies, as a part of their oversight mission ([1], [2], [3], [4], among others). Most of the effort has however been dedicated to the study of single RTGS systems, with the exception of Hellqvist and Snellman who studied the interaction between the Finnish BoF-RTGS payment system and HEXClear, the Finnish securities settlement system ([5]).

By definition, modeling system interdependencies with real data would require access to transaction data of several systems, at a transaction by transaction level. This is hard to achieve in practice due to understandable confidentiality concerns, especially on a cross-country basis where several authorities are involved.

It is therefore natural to make use of the existing theoretical models of payment and settlements systems to model system interdependencies. A simplified model of a Securities Settlement System was used by Devriese and Mitchell in [6] to investigate the spread of a liquidity crisis created by the default of the biggest participant of the system. Similarly, the approach followed in this paper relies on the use of randomly generated transactions, building on the single RTGS model developed in a previous paper by Beyeler, Glass, Bech and Soramäki ([7]).

### 2.2. Objectives of the model

A key objective was to build a model that could capture the different forms of interdependencies identified by the CPSS Working Group on System Interdependencies. In particular, the Group has identified *system-based* interdependencies (for example PvP or DvP arrangements, or liquidity bridges between two systems), *institution-based* interdependencies (when a single institution participates or provides settlement services to several systems), and *environmental-based* interdependencies (for example when a range of systems depend on a common service provider, such as a messaging service provider). The model presented in this paper explicitly incorporates the first two forms of interdependencies identified by the Working Group. Sketch 1 illustrates the different forms of interdependencies included in the model.



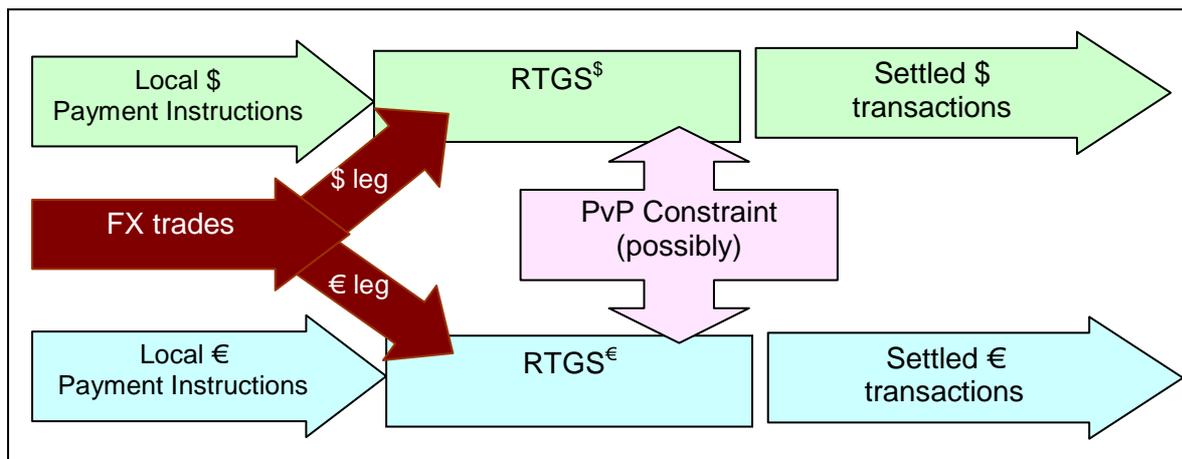
**Sketch 1: System Interdependencies**

### 3. Description of the model

#### 3.1. Model overview

An overview of the model is provided in sketch 2. The model consists of two RTGS systems settling payments in two different currencies. For the ease of exposition, these currencies are referred to dollar (\$) and euro (€) and the systems are denoted as RTGS<sup>\$</sup> and RTGS<sup>€</sup>, respectively, although the model has not been calibrated to fit any specific “real-life” situation. To simplify things, the two RTGS systems are assumed to operate continuously 24 hours a day and seven days a week. Consequently, end-of-day or overnight issues are ignored. In the model, the two RTGS systems are linked through a few “global banks” that are direct participants in both systems and carry out FX trading with each other (institution-based interdependency). Each RTGS therefore processes its own local currency payments, as well as the corresponding leg of the FX transactions traded by the global banks. Those FX legs are treated as local currency payments in each RTGS system and are thus settled one-by-one and continuously during the day.

The two RTGS systems can also be linked through a payment versus payment mechanism (system-based interdependency) that ensures the simultaneous settlement of both legs of the FX transactions on a gross basis. In the model, the PvP mechanism can be turned on (PvP) or off (non-PvP), in which case the two legs of the FX trades are settled independently.



Sketch 2: Overview of the model

The euro and dollar RTGS systems are consequently interlinked through two different channels:

- An institution-based interdependency: the common incoming flows of FX trades performed by the global banks (FX trading is made possible by the dual participation of the global banks)
- A system-based interdependency: the PvP mechanism.

With regards to local currency payments and the settlement hereof, our model is for all practical purposes similar to the single RTGS model proposed by Beyeler et al. in *Congestion and Cascades in Payment Systems* ([7]). The single RTGS model is briefly described in the next section. The model of Beyeler et al. was extended and adapted to include the settlement of FX trades among global banks. The model extension, which describes the submission and settlement of FX trades is presented in section 3.4.

## 3.2. Local payments submission and settlement

We consider an economy populated with *productive agents*, *banks*, and a *central bank* administering an interbank payment system. Figure 1 illustrates the model components, state variables, and processes, as presented in [7]. Productive agents, representing the external economy, hold deposits at banks to settle obligations arising from trades with each other. Banks maintain balances at the central bank to transfer the funds related to the payment instructions received from their agents and destined to agents banking at other banks.

A local bank  $i$  that say participates in the dollar RTGS is characterized by its level of customer deposits in dollars,  $D_i^{\$}(t)$ , and its balance of reserves at the Central Bank,  $B_i^{\$}(t)$ . A global bank is characterized by deposits and reserves in each currency. For simplicity we assume that all payments are of equal size and normalized to one. A bank's ability to execute payment instructions depends on the availability of funds on its account at the Central Bank. We assume that banks choose to settle payments whenever they have funds to do so. When a bank does not have the necessary liquidity to settle a payment (i.e., when the bank's balance at the Central Bank is zero), the payment instructions are placed on queue. Whenever funds are received by a bank, these funds are used to immediately settle previously queued instructions.

The arrival of payment instructions to the banks is modeled as a Poisson process with time varying intensity. We assume that payment instructions to a bank are driven by the level of deposits  $D_i^{\$}(t)$  held by its productive agents, which may be converted into a payment instruction with a constant probability per unit time,  $p^{\$}$ .

The expected rate of instruction arrival  $I_i^{\$}(t)$  to bank  $i$  is thus defined as:

$$\langle I_i^{\$}(t) \rangle = p^{\$} D_i^{\$}(t) \quad (1)$$

Accordingly, payment arrival rate increases as incoming payments add to deposits and decreases as payment instructions from the productive agents deplete deposits. It is important to note that the above equation provides only the average instruction arrival rate. The actual number of payment orders arriving to bank  $i$  during a time period will depend on a random draw.

## 3.3. Topology of the payment systems

### 3.3.1. Payment networks

A payment system can be seen as a network of participants linked through the payments they exchange, whereby topology refers to the structure of the payment flows. Two payment systems that would have exactly the same participants and would process similar amounts of payments for an equivalent value could still be very different in nature, depending on their topology.

In very small payment systems (such as the French payment system PNS for example, which has only 17 participants) it is common that each participant emits payments towards each of the other participants: we thus have a complete network. On the other hand, large payment systems, such as Fedwire®, are often characterized by a core of a few very large participants that

exchange many payments with many counterparties, and a large set of many very small participants that exchange a few payments with only a few counterparties.

Central banks have recently started to use the tools of network analysis to characterize the topology of their payment systems ([8], [9]). In the future, this work might help central banks to better assess the criticality of payments and participants with regard to the entire network.

With regard to modeling payment systems, it is important to ensure that the topology used in the model is realistic, as the topology plays a large role in the response of the payment system to a shock.

Both systems in the model have 100 participants out of which 94 participants in each system are “local” banks that only settle payments within that system. The remaining six participants in each system denote six “global” banks which participate in both systems. Due to the fact that they participate in both systems, they have the ability to make payments in both currencies. In the model only these banks carry out FX trading with each other. Figure 2 provides an overview of the structure of participation in the model.

### 3.3.2. Creation of the model network for local payments

Regarding local payments, the topology of both RTGS systems follows a scale-free degree distribution, meaning that both systems have many small banks which exchange few payments with a few counterparties and a few large banks which exchange many payments with many counterparties. As shown in [9, 10], real world systems such as Fedwire® and BoJ-NET can be characterized as such.

In the model, a number of links are created between the different banks to represent the payment flows. In what follows, we explain the network generation process for RTGS<sup>\$</sup> only, but the same approach also applies to RTGS<sup>€</sup>. Each bank  $i$  within RTGS<sup>\$</sup> is linked to  $K_i^{\$}$  counterparties, where the initial distribution of links per bank among the 100 participants in each network is assumed to follow a power law:

$$p(K_i^{\$}(0) = k) \propto \frac{1}{k^\gamma} \quad (2)$$

where  $\gamma$  is a parameter whose value was fitted to produce an average of 12 counterparties per participant over the 100 participants, which is representative of the average number of counterparties in the core of the Fedwire® and TARGET networks.

Payments flow in both directions along each link in the network, and only along those links. Two banks that are not connected by a link therefore exchange no payment at all. Each network link, connecting bank  $i$  and bank  $j$  is assigned two independent weights at random:  $w_{ij}^{\$}$  represents the share of bank  $i$ 's outgoing payments that are directed towards bank  $j$  and  $w_{ji}^{\$}$  represents the share of bank  $j$ 's outgoing payments that are directed towards bank  $i$ . The average payment flows between two banks need therefore not to be equal in both directions.

Although the net flow along any network link may not be zero, each RTGS system is assumed to be in equilibrium initially, that is to say that at the beginning of the simulation, each bank is expected to receive on average as many payments as it emits. The initial deposits at each bank are assigned to enforce this condition, given the randomly generated link weights.

The initial gross payment flows out of bank  $i$  in RTGS<sup>\$</sup>,  $I_i^{\$}(0)$  are on average equal to  $\langle I_i^{\$}(0) \rangle = p^{\$} D_i^{\$}(0)$ , as introduced in section 3.2. The average gross payment flows to bank  $i$  at the beginning of the simulation are  $\sum_{j \in N_i^{\$}} w_{ji}^{\$} \langle I_j^{\$}(0) \rangle = \sum_{j \in N_i^{\$}} w_{ji}^{\$} p^{\$} D_j^{\$}(0)$ , where  $N_i^{\$}$  denotes the set of banks that are linked to bank  $i$ . The initial equilibrium condition can thus be written as the following system of equations, where  $N^{\$}$  is the total number of banks in RTGS<sup>\$</sup>:

$$\forall i \in [1, N^{\$}], \quad D_i^{\$}(0) = \sum_{j \in N_i^{\$}} w_{ji}^{\$} D_j^{\$}(0) \quad (3)$$

This system of equations is then solved for the equilibrating initial deposits  $D_i^{\$}(0)$  given the specified total amount of deposits in the RTGS, and the previously chosen  $(w_{ji}^{\$})_{ji}$  coefficients.

### 3.3.3. Initial allocation of bank balances

We follow [7] on the initial allocation of the bank balances. Each participant to RTGS<sup>\$</sup> (respectively RTGS<sup>€</sup>) sets its initial central bank balance  $B_i^{\$}(0)$  (respectively  $B_i^{\€}(0)$ ) in order to control its liquidity risk (the risk of being unable to process the orders of its customers due to an insufficient balance) at the lowest possible cost (as maintaining large balances at the Central Bank entails an opportunity cost for the banks).

In this model, the initial RTGS balance of the banks is taken proportional to the square root of their initial level of deposits:

$$B_i^{\$}(0) = l^{\$} \cdot \sqrt{\frac{D_i^{\$}(0)}{d_0^{\$}}} \quad \text{and} \quad B_i^{\€}(0) = l^{\€} \cdot \sqrt{\frac{D_i^{\€}(0)}{d_0^{\€}}} \quad (4)$$

where  $l^{\$}$  and  $l^{\€}$  are parameters that characterize the level of liquidity in RTGS<sup>\$</sup> and in RTGS<sup>€</sup>, respectively, and where  $d_0^{\$}$  and  $d_0^{\€}$  are the system deposit parameters, simply taken equal to \$1 and €1 respectively.

The importance of the initial allocation of bank balances was assessed in a sensitivity study, in which different models of initial allocation were tried. It appeared that the initial allocation of liquidity between the banks does not change qualitatively the results obtained. It was also shown that for a total amount of liquidity within a RTGS, the "square root allocation" used in this paper, led to a significantly lower level of queuing than a "proportional allocation", for high levels of liquidity. This result can be intuitively related to the random walk nature of the evolution of a bank's balance ([7]).

## 3.4. FX trades submission and settlement

In addition to their participation in the two RTGS systems, the six global players make FX trades (at a constant exchange rate of 1) between each other. The local players do not participate in those FX transactions.

As for the "local payments", we assume that the customer FX transactions made by a bank are driven by the level of deposits held within this bank. The average number of dollar for euro

trades (respectively euro for dollar trades) bank  $i$  performs in a given unit of time is thus proportional to  $D_i^{\$}$  (respectively  $D_i^{\epsilon}$ ). Similarly, the probability of one of bank  $j$ 's clients engaging in a say euro for dollar trade in a given unit of time is assumed to be proportional to  $D_j^{\epsilon}$ . Therefore, if we consider that the productive agents do not have any preference regarding their FX trade counterparty, we can assume that the probability of one of bank  $i$ 's clients engaging in a dollar for euro trade with one of bank  $j$ 's clients will be proportional to the product  $D_i^{\$} D_j^{\epsilon}$ .

For every pair  $(i,j)$  of global banks, the average dollar for euro transaction rate between bank  $i$  and bank  $j$  is given by:

$$\langle I_{ij}^{\$\epsilon}(t) \rangle = p^{FX} \sqrt{\frac{D_i^{\epsilon}(0)}{D_i^{\$}(0)}} \sqrt{\frac{D_j^{\$}(0)}{D_j^{\epsilon}(0)}} D_i^{\$}(t) D_j^{\epsilon}(t) \quad (5)$$

where  $p^{FX}$  is a constant parameter describing the level of FX trading activity between the two RTGS systems. The use of the  $\sqrt{\frac{D_i^{\epsilon}(0)}{D_i^{\$}(0)}} \sqrt{\frac{D_j^{\$}(0)}{D_j^{\epsilon}(0)}}$  proportionality coefficient guarantees that  $\langle I_{ij}^{\$\epsilon}(0) \rangle = \langle I_{ji}^{\$\epsilon}(0) \rangle$  as well as a finite return time towards the initial steady state. The retained proportionality coefficient simply translates the fact that we expect certain stability regarding the currency holdings of the banks during a simulation. As in reality, we do not expect the largest participant to RTGS<sup>€</sup> selling off all its euros in order to become the largest participant in RTGS<sup>€</sup>. The FX trading activities of the global players will thus only let them oscillate around their starting position.

Contrary to the case of local payments, we chose to describe the FX market as a complete network, that is to say a system where each participant trades with every other participant. This assumption is fairly realistic for a small system of six large banks of similar size, while it would have been totally unrealistic for a local RTGS system with many participants of different sizes.

## 4. Correlations between the two systems

In this section, we wish to investigate whether the settlement activity of the two RTGS systems becomes correlated because of the two system interdependencies introduced in the model (the PvP mechanism and the dual participation of the global players). We consider that the settlement activity of the two RTGS systems is (positively) correlated provided that, statistically, a period of high settlement activity (respectively a period of low settlement activity) within one system corresponds to a period of high settlement activity in the other system (respectively a period of low settlement activity).

We can observe visually the degree of correlation between the two systems by using settlement rate scatter plots such as the ones presented in figure 3 and figure 4. Two simulations were performed to make each of those two figures. One simulation was run with a low level of liquidity (blue dots), and one simulation was run with a high level of liquidity (red dots). Each dot of the scatter plot corresponds to a certain time window of the simulation (the duration of the simulation was divided into one thousand time windows of constant duration). The abscissa of the dot corresponds to the settlement rate observed in RTGS<sup>€</sup> during the considered time window (i.e., the number of local payments and FX legs settled in RTGS<sup>€</sup> divided by the duration of the

time window). The ordinate of the dot corresponds to the settlement rate observed in RTGS<sup>\$</sup> during the same time window.

In both figure 3 (non-PvP settlement of FX trades) and figure 4 (PvP settlement), we can observe that the amplitude of the variations of the settlement rates is much higher at low liquidity. Indeed, at high liquidity, the payments are settled nearly immediately. As a consequence, the queues are almost empty and the settlement rate remains very close to the arrival rate of the payment orders. At low liquidity however, the size of the queues vary greatly over time. Periods of congestion, characterized by a low settlement rate and the building up of the queues, alternate with periods of cascades, characterized by a high settlement rate and a massive release of queued payments.

With regard to the observed degree of correlation of the two systems, table 1 summarizes the main findings of figure 3 and figure 4.

Degree of correlation between the settlement rates of the two systems		Settlement mechanism for FX transactions	
		non-PvP	PvP
Level of liquidity (the same in both systems)	Low	-0.02	0.83
	High	0.22	0.22

**Table 1: Degree of correlation between the settlement rates of the two systems (a value of 0 corresponds to a perfectly uncorrelated case, while a value of 1 corresponds to a perfectly correlated case)**

At high liquidity, there is a slight degree of correlation between the two systems, corresponding to the level of FX trading. This was expected since a period of high FX trading will tend to increase simultaneously the throughput in both systems. The settlement mechanism (PvP or non-PvP) does not have any impact on the results at high liquidity, as all payments settle nearly immediately, irrespective of the settlement mechanism in place. The degree of correlation between the outputs of the two systems is 0.22, both in the PvP case and in the non-PvP case. This value tends to increase when the level of FX activity (the relative share of FX trades compared to the total amount of payments processed) increases. The top sketch of figure 5 illustrates the coupling induced by the FX trading activity at high liquidity.

At low liquidity, the systems are no longer governed by the arrival of payment orders, but rather by their internal physics of congestion (the payment orders are queued due to a lack of liquidity) and cascades (as the settlement of a newly arrived payment order can trigger the release of several queued payments). The two systems then appear completely uncorrelated in the non-PvP case, as the correlation caused by the common FX input has disappeared in the internal process of congestion and cascades. The scatter plot shown in figure 3 has thus a nearly perfect circular shape. The middle sketch of figure 5 illustrates the decoupling of the two systems.

At low liquidity in the PvP case, the settlement rates of the two systems appear highly correlated, as shown by the “comet shape” of the scatter plot presented in figure 4. The correlation caused by the common FX input in the high liquidity case has been replaced by a mechanical PvP release correlation between the two systems. The degree of correlation of the settlement rates of the two systems is then 0.83. The bottom sketch of figure 5 illustrates how the PvP mechanism creates a coupling between the two systems at low liquidity.

## 5. FX settlement risk under non-PvP

In this chapter, we will show that in the non-PvP case, the credit exposures that arise between the global players create a strong interdependency between the two systems. The level of exposures will be shown to depend on the liquidity available in each of the two systems and to increase as the liquidity is decreased (section 5.2). Moreover, we will demonstrate that a structural imbalance between the two systems in terms of liquidity can have the same effects as a time zone difference between the two systems, and thus result in significantly high levels of exposure (section 5.3). Finally, we will observe that credit exposures can be drastically reduced by granting the FX transactions a higher level of priority than the local payments (section 5.4).

### 5.1. Calculation of the FX exposures

When the FX trades are settled non-PvP, the bank that pays the first leg of the transaction will bear a FX credit risk until the other leg of the transaction is settled in the other RTGS. Sketch 3 introduces the concept of time-averaged exposure, defined as the product of the amount of credit risk involved by the duration of the exposure. The exposure thus corresponds to the area of the colored rectangles in sketch 3.

An attempt at quantifying those exposures was made using the proposed model and several simulations were thus run in the non-PvP case, with varying levels of liquidity in the two RTGS systems.

In the non-PvP case, we define the following indicators:

- The time-averaged gross exposure of the dollar selling banks to the euro selling banks

$$Exposure_{\$/\text{€selling}} = \sum_k \text{Value}_k \cdot \max(0; t_k^\text{€} - t_k^\text{\$/}) \cdot \frac{1}{T} \quad (6)$$

- The time-averaged gross exposure of the euro selling banks to the dollar selling banks

$$Exposure_{\text{€selling}/\$} = \sum_k \text{Value}_k \cdot \max(0; t_k^\text{\$/} - t_k^\text{€}) \cdot \frac{1}{T} \quad (7)$$

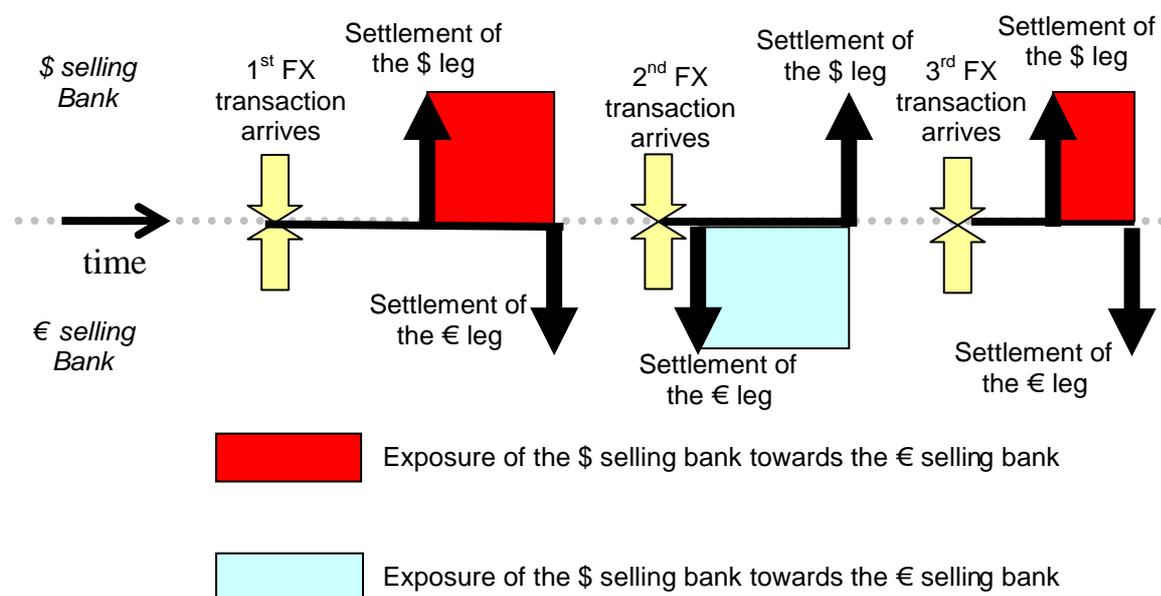
where:

- The sum is done over all the FX transactions  $k$  settled during the considered period
- $T$  is the duration of the considered period
- $\text{Value}_k$  refers to the value of transaction  $k$  (in this paper, it is always equal to 1)
- $t_k^\text{€}$  is the settlement time of the euro leg of transaction  $k$
- $t_k^\text{\$/}$  is the settlement time of the dollar leg of transaction  $k$

The equations above simply reflect the fact that, in a FX transaction, the dollar selling bank will be facing an exposure towards the euro selling bank, if the euro leg of the transaction settles after the dollar leg of the transaction (i.e., if  $t_k^\text{€} > t_k^\text{\$/}$ ).

It is important to keep in mind that we only consider gross exposures here. Let's consider the case where the two opposite transactions, transaction 1 (bank  $i$  sells \$1 for €1 to bank  $j$ ), and transaction 2 (bank  $j$  sells \$1 for €1 to bank  $i$ ) are submitted simultaneously and where the euro leg of both transaction 1 and transaction 2 settle, while both dollar legs remain pending. The euro

selling banks are then exposed to the dollar selling banks for a value of \$2, while the net exposure of bank  $i$  towards bank  $j$  would be zero.



**Sketch 3: Exposures created by the non-PvP settlement of FX transactions**

## 5.2. Exposures with the same level of liquidity in both systems

The proposed model was run to quantify the gross credit exposures resulting from the FX transactions in the non-PvP case for various levels of liquidity. We first investigate the case where both systems have the same level of liquidity. The results are presented in figure 6 and the main results are sum-up in table 2. It is not surprising to observe that the credit exposures increase sharply when the liquidity is decreased. At high levels of liquidity, both legs of the FX transactions settle nearly instantly and thus the related credit exposures remain very limited.

		Average gross exposure of the \$ selling banks to the € selling banks	Average gross exposure of the € selling banks to the \$ selling banks	Total exposures
Level of liquidity (the same in both systems)	Lowest	734	676	1410
	Low	376	381	757
	High	221	231	452
	Highest	15.3	13.7	29

**Table 2: Gross exposures in the non-PvP case, as a function of the level of liquidity in both systems, with a normal priority for FX payments and a high level of FX activity**

## 5.3. Exposures with different levels of liquidity in the two systems

It is well known that time zone differences between RTGS systems result in such systematic exposures for non-PvP FX trades. In a somehow similar way, when one system (for example the euro RTGS) has a significantly higher level of liquidity than the other system, the euro leg of the FX transactions will settle significantly faster than the dollar leg. As a consequence, the banks that are selling euro for dollar can expect to face a much higher credit risk than normal.

This phenomenon is illustrated figure 7, and the main results are sum-up in table 3.

		Average gross exposure of the \$ selling banks to the € selling banks	Average gross exposure of the € selling banks to the \$ selling banks	Total exposures
Level of liquidity in the \$ RTGS	Lowest	0.0377	3150	3150
	Low	0.413	1400	1400
	High	8.53	365	374

**Table 3: Gross exposures in the non-PvP case, as a function of the level of liquidity in RTGS<sup>\$</sup>, for a constant very high level of liquidity in RTGS<sup>€</sup>.**

A comparison of table 2 with table 3 teaches us that when the liquidity in RTGS<sup>\$</sup> is maintained constant at the lowest level, increasing the liquidity in RTGS<sup>€</sup> from the lowest level to a very high level, increases the total exposures from 1410 to 3150.

A similar phenomenon was also observed when the average settlement delay within a currency zone was decreased thanks to an efficient intraday liquidity market (refer to [7] for a description of the model retained to describe the operation of a liquidity market), while the other currency zone was characterized by a low liquidity level.

#### 5.4. Influence of FX transaction priority

The influence of the chosen priority level for the FX transactions was also investigated. In the model, the two legs of the FX transactions can either be given a higher priority than the local payments (in that case, when a global player lacking liquidity receives a payment, the received liquidity will only be used to settle a local payment if there is no pending outgoing FX leg to settle) or an equal priority (in that case, the transactions are settled according to their order of arrival, irrespectively of their nature). Box 8 provides a comparison of the situation between the high priority case (figure 8.2) and the normal priority case (figure 8.1). The simulations clearly show that using a higher priority for FX payments than for local payments sharply decreases the overall level of credit risk.

Table 4 sums-up the main results of figure 8.2 and should be compared with table 2. It appears that the exposures have been decreased enormously by giving a high priority to the FX transactions. In addition, we can note that the magnitude of the decrease is highest for the intermediate values of the liquidity level.

		Average gross exposure of the dollar selling banks to the euro selling banks	Average gross exposure of the euro selling banks to the dollar selling banks	Total exposures
Level of liquidity (the same in both systems)	Lowest	16.8	16.4	33.2
	Low	4.49	4.35	8.84
	High	2.71	2.78	5.49
	Highest	0.384	0.362	0.746

**Table 4: Gross exposures in the non-PvP case, as a function of the level of liquidity in both systems with high priority given to FX instructions, for a high level of FX activity**

## 6. Queuing under non-PvP and PvP

In this chapter, we investigate the impact of liquidity on the level of queuing, this time for the considered case of two RTGSs interacting through FX transactions. We will show that the PvP mechanism introduces a strong interdependency between the two systems that tends to increase the average level of queuing when both systems have the same level of liquidity (section 6.1). In addition, we will prove that, unlike in the non-PvP case, where the level of queuing within one system only depends on the liquidity available within this system, when the FX transactions are settled PvP, the level of queuing within one RTGS also becomes dependent on the liquidity present within the other system (section 6.2). We will also show that this effect increases with the level of FX activity (section 6.3), and sharply decreases when the FX trades are given a higher order of priority than the local payments (section 6.4).

### 6.1. Queuing with the same level of liquidity in both systems

We first investigate the case where both systems have the same level of liquidity. Figure 9 shows the average number of queued payments in the two RTGS systems, as a function of the level of liquidity in the two systems. The first obvious observation is that the level of queuing increases as the liquidity decreases, whether PvP is used or not. We can also notice that, as the level of liquidity is decreased in the two systems, the scatter plots become more dispersed, which shows that as the size of the queues increases, the amplitude of their variations over time also increase.

With regard to the influence of the PvP mechanism on the average size of the queues, table 5 sums up the observations of figure 9. It appears that in those conditions, the use of PvP settlement increases the average level of queuing (and therefore increases the average settlement delay) in both systems when both systems have a low level of liquidity.

Average queue in RTGS <sup>§</sup> (left ) and in RTGS <sup>€</sup> (right )		Settlement mechanism for FX transactions			
		non-PvP		PvP	
Level of liquidity (the same in both systems)	Lowest	33 100	33 400	35 300	35 300
	Low	14 500	14 600	15 700	15 400
	High	4 510	4 480	4 890	4 900
	Highest	240	241	255	253

**Table 5: Average number of queued payments in both systems as a function of liquidity level and of the chosen settlement mechanism when both systems have the same level of liquidity.**

We can complement this analysis by looking at table 6 that provides the standard deviation of the settlement rate in the simulated cases. As expected, the use of PvP mechanism increases the variability of the settlement rate. We can also note that the tendency of PvP to increase settlement rate variability is strongest at intermediate liquidity levels.

Standard deviation of settlement rate in RTGS <sup>§</sup> (left ) and in RTGS <sup>€</sup> (right )		Settlement mechanism for FX transactions			
		non-PvP		PvP	
Level of liquidity (the same in both systems)	Lowest	1 950	1 930	2 150	2 120
	Low	690	697	1 160	1 150
	High	230	232	377	380
	Highest	116	116	117	117

**Table 6: Standard deviation in settlement rate in both systems as a function of liquidity level and of the chosen settlement mechanism when both systems have the same level of liquidity.**

## 6.2. Queuing with different levels of liquidity in the two systems

### 6.2.1. Without a PvP mechanism

This time, we investigate the consequences of a structural liquidity imbalance between the two systems. As a convention, we set the liquidity of dollar system to a lower level than the liquidity of euro system, and we observe how the level of queuing in the two systems evolve as we let the liquidity level within the two systems vary. Figure 10 shows the obtained results for various levels of liquidity as scatter plots. It appears that the liquidity contrast between the two RTGS systems create systematic differences in queuing between the richer (higher liquidity) and poorer (lower liquidity) system.

Table 7 sums up the main results of figure 10. As expected, the average size of the queue increases sharply for a given system when liquidity within this system is decreased. We can also note that the average size of the queue of a system depends on the level of liquidity available in the other system.

Average queue in RTGS \$ (left) and in RTGS € (right). The numbers between brackets are the standard deviation of the queue.		Level of liquidity in the RTGS <sup>\$</sup>							
		Lowest		Low		High		Highest	
Level of liquidity in RTGS <sup>€</sup>	Lowest	33 100	33 400						
	Low	33 400	14 600	14 500	14 600				
	High	32 600	4 440	14 600	4 460	4 510	4 480		
	Highest	32 900	235	14 800	241	4 500	238	240	241

**Table 7: Average number of queued payments in both systems in the non-PvP case as a function of the level of liquidity in RTGS € and in RTGS \$, for a high level of FX activity and a normal priority of FX payments.**

This conclusion is confirmed by table 8 which presents the standard deviation of the settlement rate in the two systems. It clearly appears that the variability of the settlement rate within a system does not depend on the level of liquidity available in the other system.

Standard deviation of settlement rate in RTGS <sup>\$</sup> (left ) and in RTGS <sup>€</sup> (right )		Level of liquidity in the RTGS <sup>\$</sup>							
		Lowest		Low		High		Highest	
Level of liquidity in RTGS <sup>€</sup>	Lowest	1 950	1 930						
	Low	1 940	695	690	697				
	High	1 940	233	709	230	230	232		
	Highest	1 900	117	701	116	233	116	116	116

**Table 8: Standard deviation of settlement rate both systems in the non-PvP case, as a function of the level of liquidity in RTGS € and in RTGS \$, for a high level of FX activity and a normal priority of FX payments.**

**We can therefore conclude that in the non-PvP case, the average level of queuing in a system as well as the variability of its settlement rate, is determined only by the liquidity present in that system.**

## 6.2.2. With the PvP mechanism

The simulations conducted in section 6.2.1 were re-made, this time assuming that the FX transactions are settled using a PvP mechanism. Figure 11.1 shows the average level of queuing in the two systems, as a function of the level of liquidity in RTGS<sup>\$</sup> and in RTGS<sup>€</sup>. Figure 11.1 and figure 10 differ only by the chosen settlement mechanism (non-PvP for figure 10, and PvP for figure 11.1), and a comparison between those two clearly highlights the influence of the PvP mechanism. Especially, when the liquidity level is high in RTGS<sup>€</sup> and low in RTGS<sup>\$</sup>, a further reduction of the liquidity level in RTGS<sup>\$</sup> increases significantly the level of queuing in RTGS<sup>€</sup> in the PvP case (figure 11.1), while it remains without effect in the non-PvP case (figure 10).

Table 9 sums up the main results provided by figure 11.1. For each level of liquidity in the two systems, the table presents the average number of queued payments in each RTGS. Of course, the average size of the queue in a given system increases sharply when liquidity within this system is decreased, as in the non-PvP case. Contrary to the non-PvP case however, the average size of the queue in one system also depends on the liquidity available in the other system.

Average queue in RTGS <sup>\$</sup> (left ) and in RTGS <sup>€</sup> (right)		Level of liquidity in the RTGS <sup>\$</sup>							
		Lowest		Low		High		Highest	
Level of liquidity in RTGS <sup>€</sup>	Lowest	35 300	35 300						
	Low	33 400	18 100	15 700	15 400				
	High	33 400	10 700	14 800	5 890	4 890	4 900		
	Highest	32 400	3 600	14 500	1 670	4 580	618	255	253

**Table 9: Average number of queued payments in both systems in the PvP case, as a function of the level of liquidity in RTGS<sup>€</sup> and in RTGS<sup>\$</sup>, for a high level of FX activity and a normal priority of FX payments.**

More specifically, when liquidity is decreased within the “less liquid” system, the level of queuing increases significantly within the “more liquid” system. This effect appears especially strong for intermediate levels of liquidity in the “more liquid” system. In addition, we also observe that the level of queuing in the “less liquid” system decreases when the liquidity is increased in the “more liquid” system.

Table 10 presents the standard deviation of the settlement rate in both systems, in the same conditions. We can observe that the PvP mechanism has an impact on the variability of the settlement rate by comparing table 10 with table 8. The variability of the settlement rate within one system becomes dependent on the other system's liquidity when FX trades are settled PvP, yet the effect of the other system's liquidity is varying, unlike what we observe for the average queues. A detailed analysis of this effect will require further investigation.

Standard deviation of settlement rate in RTGS \$ (left ) and in RTGS € (right)		Level of liquidity in the RTGS <sup>\$</sup>							
		Lowest		Low		High		Highest	
Level of liquidity in RTGS <sup>€</sup>	Lowest	2 150	2 120						
	Low	2 000	714	1 160	1 150				
	High	1 990	388	724	312	377	380		
	Highest	1 900	323	694	166	234	126	117	117

**Table 10: Standard deviation of settlement rate in both systems in the PvP case, in the PvP case, as a function of the level of liquidity in RTGS € and in RTGS \$, for a high level of FX activity and a normal priority of FX payments.**

We can therefore conclude that in the PvP case, the average level of queuing in one RTGS, as well as the variations of its settlement rate, do not depend only on the level of liquidity available in that given RTGS, but also on the level of liquidity present in the other RTGS. The two systems therefore appear interlinked as an increase in the level of liquidity in one system either through a change in its participant's behavior or through a change in the Central Bank policy will create a positive externality for the other system.

### 6.3. Influence of the level of FX activity

The level of FX activity, i.e. the relative volume of FX transactions with regard to the local payments, is a parameter of the presented model. The aim of this short section is to investigate to which extent the liquidity interdependency created by the PvP mechanism discovered in section 6.3.1 will be dependent on the level of FX activity. Box 11 provides a comparison between the situation observed for a high level of FX activity (figure 11.1) and the results obtained for a low level of FX activity (figure 11.2). As could be expected, we notice that the higher the level of FX activity, the stronger the interdependency becomes between the two systems linked by the PvP mechanism.

The results presented in figure 11.2 are recalled in table 11. The average level of queuing in the PvP case for a low level of FX activity, appears somewhat similar to the results obtained in the non-PvP case (table 7). The level of queuing within a system appears to depend only very weakly on the other system's level of liquidity. However, when the level of liquidity is maintained to its highest value in RTGS<sup>€</sup>, the level of queuing in RTGS<sup>€</sup> seem to be still significantly affected by the level of liquidity in RTGS<sup>\$</sup>.

Average queue in RTGS <sup>\$</sup> (left) and in RTGS <sup>€</sup> (right)		Level of liquidity in the RTGS <sup>\$</sup>							
		Lowest		Low		High		Highest	
Level of liquidity in RTGS <sup>€</sup>	Lowest	33 500	33 700						
	Low	32 400	15 200	15 000	14 700				
	High	33 300	4 800	14 800	4 810	4 680	4 570		
	Highest	32 700	810	14 700	476	4 580	303	238	246

**Table 11: Average number of queued payments in both systems in the PvP case, as a function of the level of liquidity in RTGS<sup>€</sup> and in RTGS<sup>\$</sup>, for a low level of FX activity, and a normal priority of FX payments**

The lack of strong interlinkage between the two systems is confirmed by table 12 which presents the standard deviation of the settlement rate in both RTGS systems.

Standard deviation of settlement rate in RTGS <sup>\$</sup> (left) and in RTGS <sup>€</sup> (right)		Level of liquidity in the RTGS <sup>\$</sup>							
		Lowest		Low		High		Highest	
Level of liquidity in RTGS <sup>€</sup>	Lowest	1 580	1 600						
	Low	1 650	590	661	633				
	High	1 690	212	622	239	231	231		
	Highest	1 660	112	611	107	214	107	107	108

**Table 12: Standard deviation of settlement rate in both Systems in the PvP case, as a function of the level of liquidity in RTGS<sup>€</sup> and in RTGS<sup>\$</sup>, for a low level of FX activity, and a normal priority of FX payments**

## 6.4. Influence of FX transaction priority

In the model, the two legs of the FX transactions can either be given a higher priority than the local payments (in that case, when a global player lacking liquidity receives a payment, the received liquidity will only be used to settle a local payment if there is no pending outgoing FX leg to settle), or a normal priority (in that case, the transactions are settled according to their order of arrival, irrespectively of their nature). Box 12 provides a comparison between the normal priority case (figure 12.1), and the high priority case (figure 12.2). It clearly appears that imposing a high priority for FX payments drastically reduces the dependency of one RTGS on the other RTGS's liquidity.

Table 13 sums up the results of figure 12.2. The average level of queuing in the PvP case for a high FX priority (table 13), appears very similar to the results obtained in the non-PvP case (table 7). The level of queuing within a system appears fairly independent of the other system's level of liquidity.

Average queue in RTGS \$ (left) and in RTGS € (right)		Level of liquidity in the RTGS <sup>\$</sup>							
		Lowest		Low		High		Highest	
Level of liquidity in RTGS <sup>€</sup>	Lowest	35 000	34 900						
	Low	33 200	15 800	15 300	15 300				
	High	32 900	4 720	14 900	4 720	4 760	4 750		
	Highest	33 800	286	14 500	268	4 500	240	241	247

**Table 13: Average number of queued payments in both systems in the PvP case, as a function of the level of liquidity in RTGS<sup>€</sup> and in RTGS<sup>\$</sup>, for a high level of FX activity rate and a high FX priority**

Table 14 presents the standard deviation of the settlement rate in both systems, in the same conditions. We observe that the variability of the settlement rate in the PvP case with a high level of priority for the FX payments is significantly higher than in the non-PvP case (table 8). The importance of this effect depends however greatly on the level of liquidity available. Further investigation will be required to provide a definitive explanation of the phenomena involved.

Standard deviation of settlement rate in RTGS \$ (left) and in RTGS € (right)		Level of liquidity in the RTGS <sup>\$</sup>							
		Lowest		Low		High		Highest	
Level of liquidity in RTGS <sup>€</sup>	Lowest	2 530	2 530						
	Low	2 170	1 340	1 350	1 360				
	High	1 800	615	892	691	451	454		
	Highest	1 690	128	619	119	227	117	116	117

**Table 14: Standard deviation of settlement rate in both systems in the PvP case, as a function of the level of liquidity in RTGS<sup>€</sup> and in RTGS<sup>\$</sup>, for a high level of FX activity rate and a high FX priority**

## 7. Conclusion

The parsimonious model of RTGS payment system previously developed and presented in [7] has been used to describe the interactions between two separate systems, each operating in a distinct currency. The original model has been slightly modified and complemented by a simple model describing the random arrival of FX transactions performed by a set of global banks that participate in both systems.

This dual participation, and the resulting common inflow of FX trades, creates an institution-based interdependency between the two systems. As a result, the activity of the two systems is shown to become correlated at high levels of liquidity, in the sense that a period of high settlement rate within one RTGS is statistically likely to correspond to a period of high settlement rate within the other RTGS.

In the model, FX trades are settled on a gross basis, either PvP (both legs of the FX transactions can only be settled simultaneously) or non-PvP (both legs of the FX transactions are settled independently). The use of a PvP mechanism to settle FX trades results in a system-based interdependency between the two systems. Consequently, the activity of the two systems is shown to become correlated at low levels of liquidity.

When the FX trades are settled non-PvP, some credit exposures are created between the global banks that engage in FX trading. Those exposures are shown to be dependent on the level of liquidity present in each RTGS. Moreover, it appears that a structural liquidity imbalance between the two systems leads to very high exposures, by acting in a similar way as a time zone difference between the two systems. The model however shows that those exposures can be drastically reduced by granting the FX transactions a higher level of priority than the local payments.

When the FX trades are settled PvP, the credit exposures between the global banks vanish. However, the PvP mechanism creates another kind of interdependency between the two systems. Indeed, the model shows that in the PvP case, the average level of queuing within one RTGS does not depend only on its own level of liquidity like in the non-PvP case, but also on the level of liquidity in the other RTGS. More specifically, when liquidity is decreased within the “less liquid” system, the level of queuing increases significantly within the “more liquid” system. This effect appears especially strong for intermediate levels of liquidity in the “more liquid” system. In addition, we also observe that the level of queuing in the “less liquid” system decreases when the liquidity is increased in the “more liquid” RTGS. This interdependency increases with the level of FX activity, and sharply decreases when the FX trades are given a higher order of priority than the local payments.

The results obtained so far by the model can already be used to qualitatively describe and document the effect of the interdependency created by the FX transactions and the possible PvP mechanism on the activity of the two systems. In the future, the model could be used to investigate more specific questions, such as the consequences of net funding for the settlement of FX transactions, or the impact of the creation of an intraday FX swap market. The cross-border spread of liquidity disruptions, for example following the technical default of a major participant, could also be modeled with the proposed approach.

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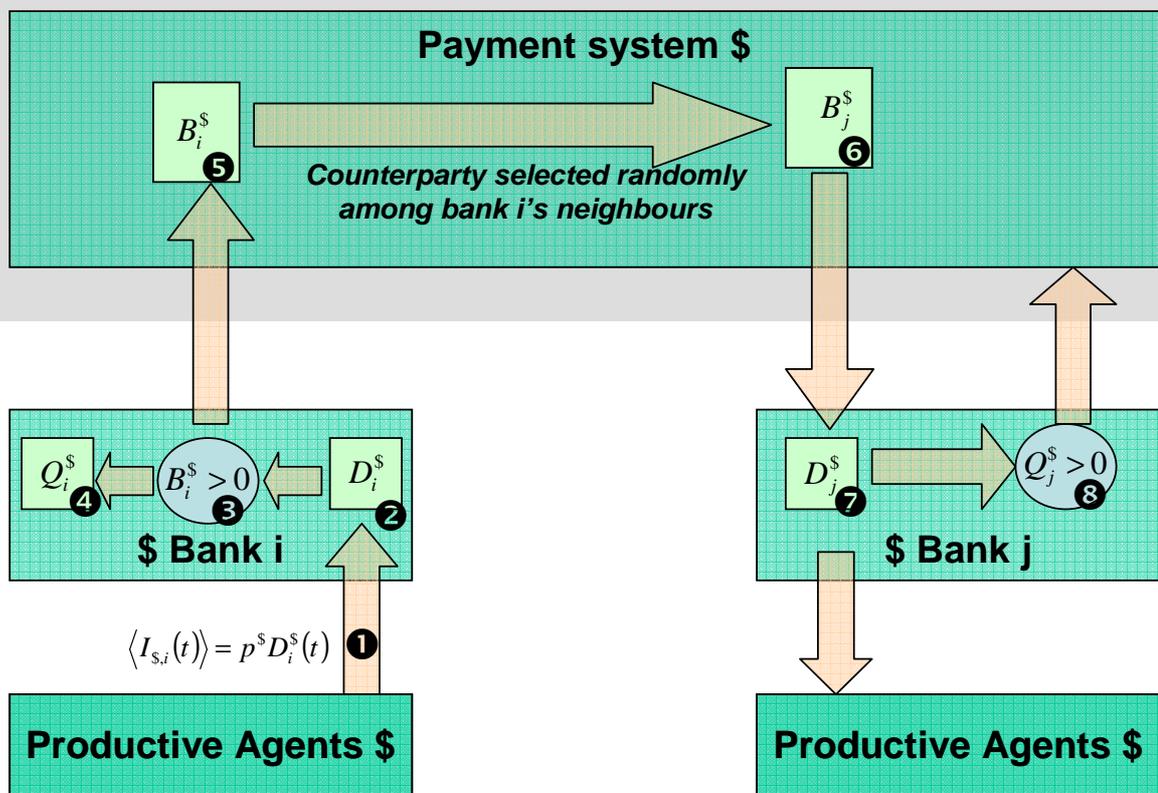
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## **Table of symbols:**

The variables relative to the local payments were only explicitly provided for RTGS<sup>\$</sup>.

Variable	Dimension	Description
$B_i^s(t)$	money (\$)	Payments account balance of Bank $i$ within RTGS \$
$d_0^s$	money (\$)	System deposit size parameter in RTGS \$, taken equal to \$1
$D_i^s(t)$	money (\$)	Total amount of \$ deposits held by Bank $i$ on behalf of its customers at time $t$
$I_i^s(0)$	1/time	Rate of arrival of payment instructions to Bank $i$ in RTGS \$
$I_{ij}^{s\epsilon}(t)$	1/time	Rate of arrival of FX trades instructions consisting of Global Bank $i$ selling \$1 to Global Bank $j$ for €1
$K_i^s$	–	Number of counterparties of Bank $i$ in RTGS \$
$l^s$	money (\$)	Liquidity factor parameter in RTGS \$
$N^s$	–	Total number of banks in RTGS \$
$N_i^s$	–	Number of counterparties of Bank $i$ within RTGS \$
$p^s$	1/(money (\$).time)	Probability that a payment instruction will be issued in RTGS \$ per unit of time and of deposit
$p^{FX}$	1/(money (\$).money (€).time)	Probability that a payment instruction will be issued in RTGS \$ per unit of time and of deposit
$w_{ij}^s$	–	Share of Bank $i$ 's outgoing payments that are directed towards Bank $j$ in RTGS \$
$\gamma$	–	Power-law exponent of the distribution of counterparties per bank. Its value was fitted so as to produce an average of 12 counterparties per bank

## Central bank \$



- ❶ Bank i receives a continuous stream of payment orders from its depositors. The average volume of payment orders received by a bank is taken as proportional to the current level of deposits at this bank.
- ❷ Depositor account of bank i,  $D_i^s$  is debited.
- ❸ The RTGS account balance of bank i,  $B_i^s$ , is checked.
- ❹ If Bank i does not have sufficient liquidity at the Central Bank to settle the payment, (since we consider only payments of unit size, we just check if  $B_i^s$  is greater than zero), the payment is queued.
- ❺ Otherwise, the payment is settled and  $B_i^s$  is decremented.
- ❻ The receiving bank is taken randomly among Bank i's counterparties. The RTGS account of the receiving bank, bank j, is incremented.
- ❼ The depositor account of bank j is incremented. The probability of bank j to receive a payment order from one of its depositors is thus mechanically increased.
- ❽ If bank j has some outgoing queued payments waiting, the payment with the earliest submission time is released (FIFO order).

Fig 1: Processing of local payments

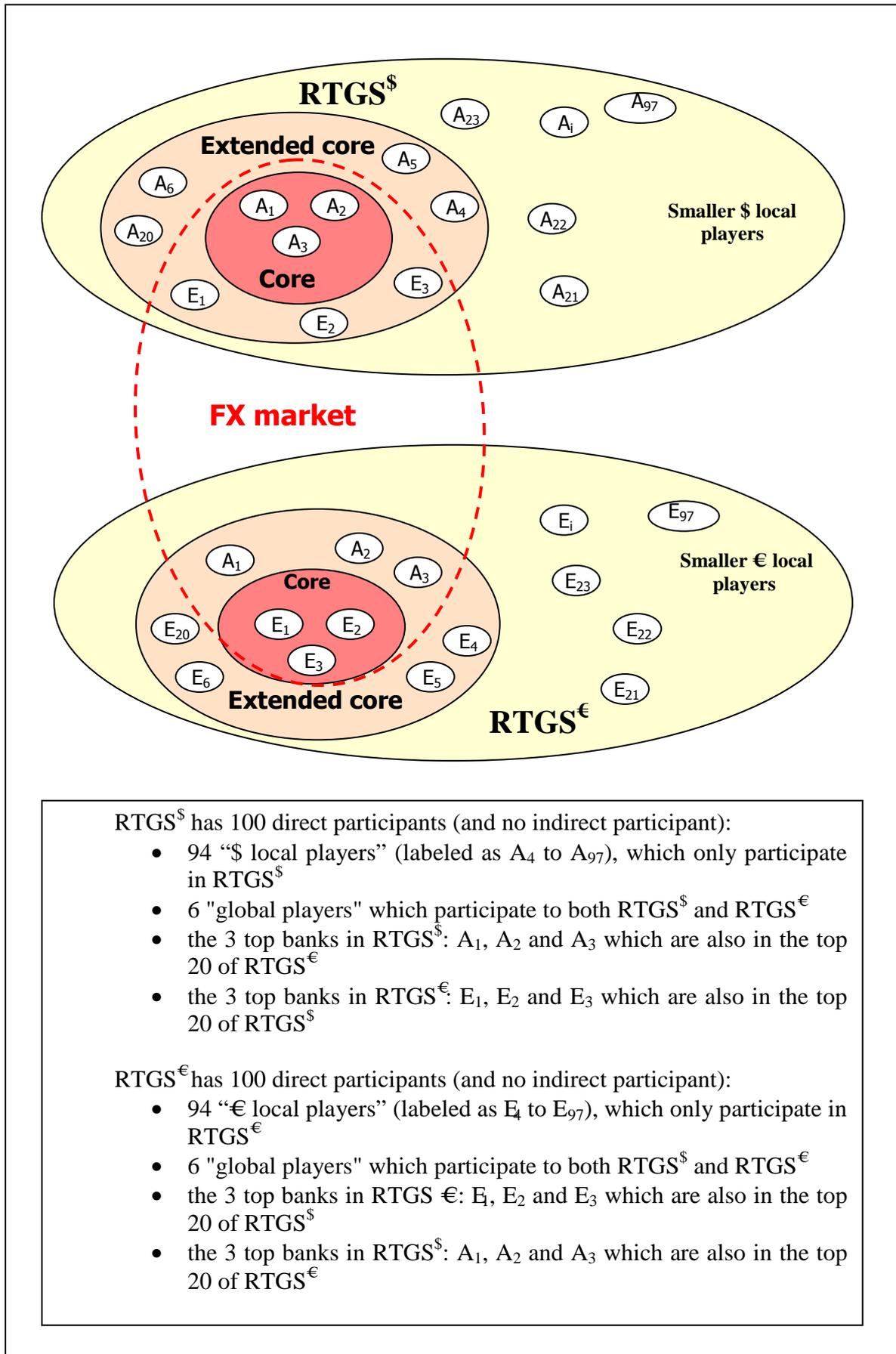


Fig 2: Structure of participation in the model

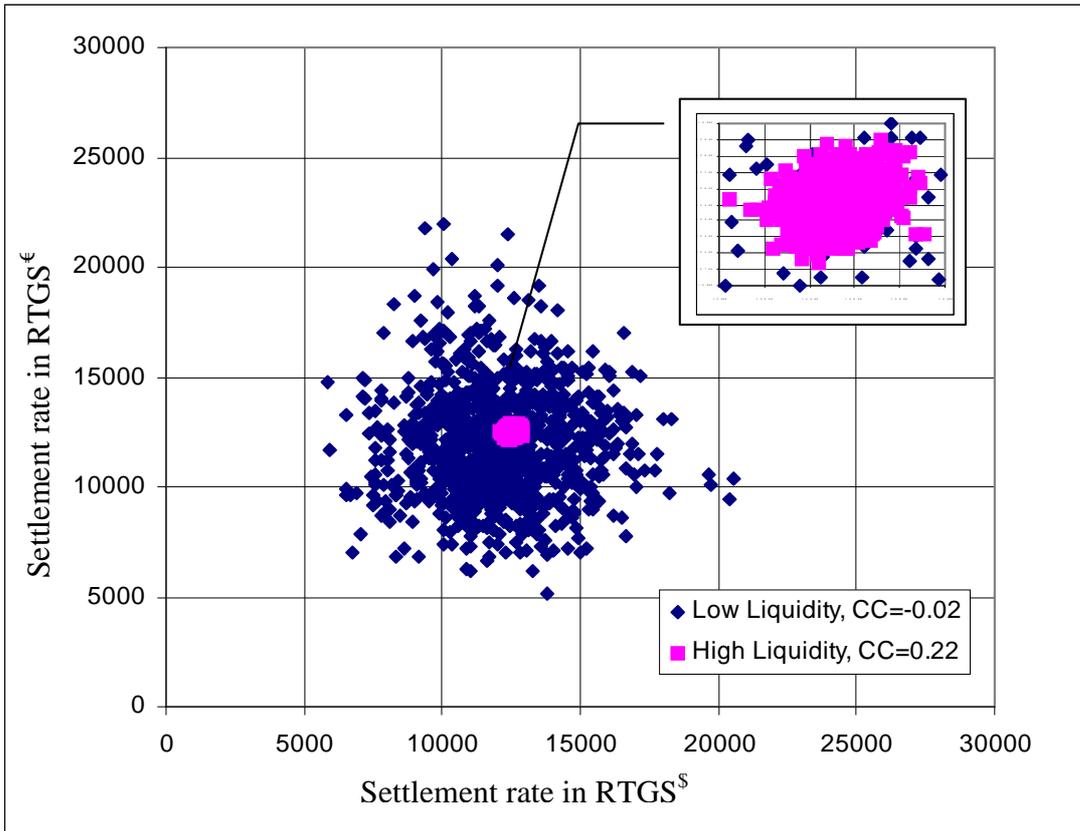


Fig 3: Correlation of the settlement rates in the two RTGSs, non-PvP case

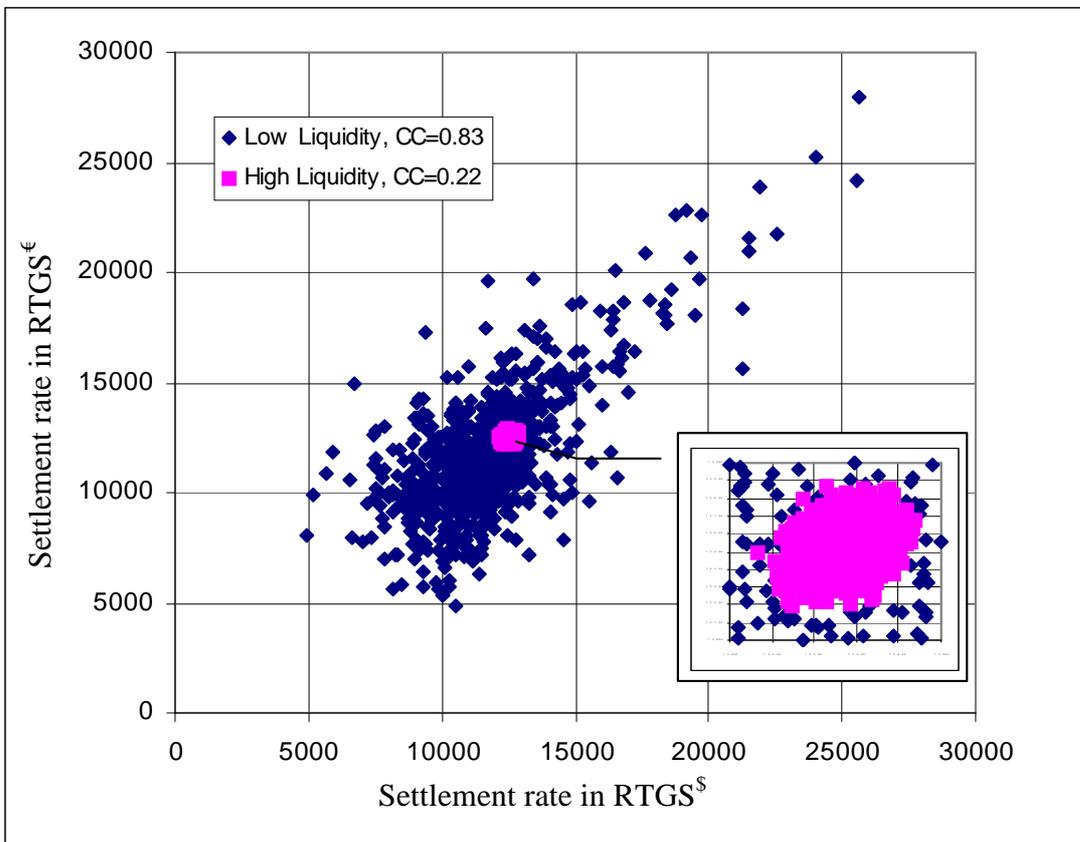
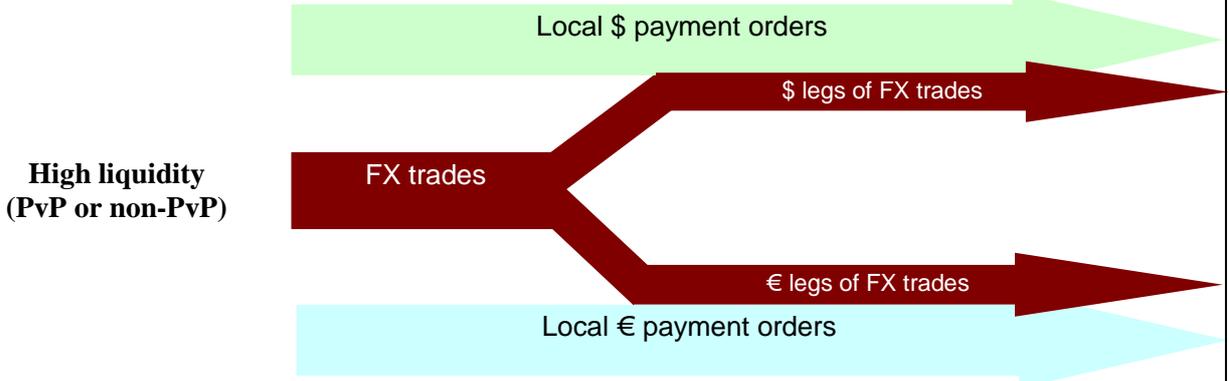
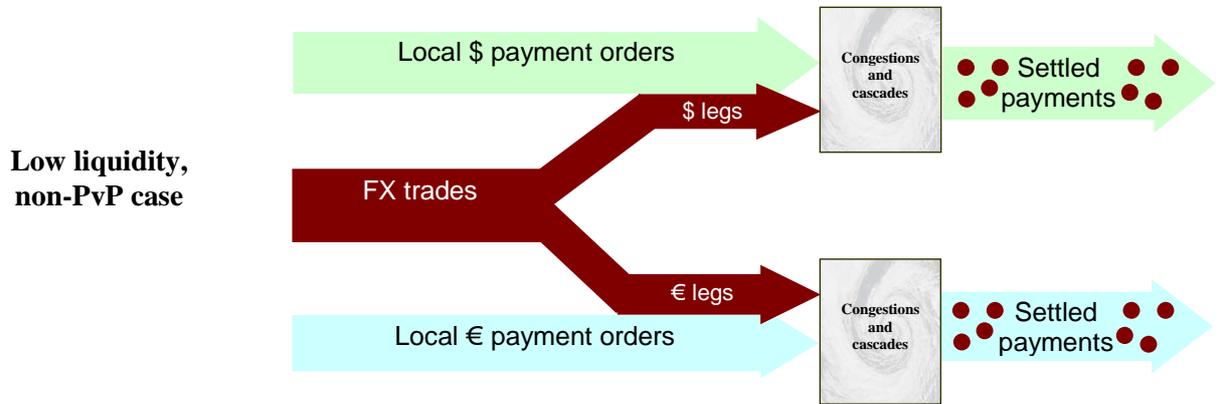


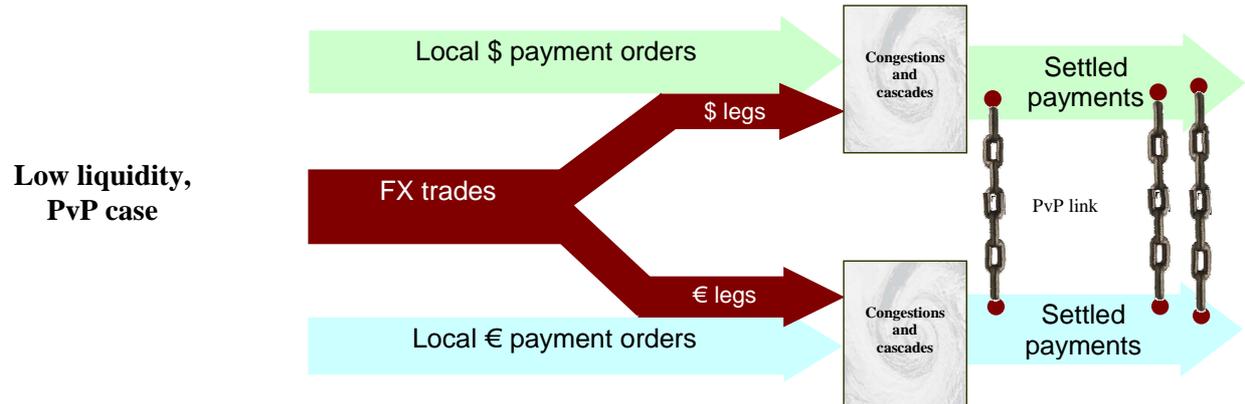
Fig 4: Correlation of the settlement rates in the two RTGSs, PvP case



At high liquidity (PvP or non-PvP), transactions settle nearly instantly after their submission. The two legs of the FX transactions that are submitted simultaneously to both RTGSs, will settle nearly simultaneously at high liquidity. Therefore the output of the two RTGSs will be correlated, and the amount of correlation between the outputs will increase with the relative importance of FX trading compared to local payments. The settlement mechanism (PvP or non-PvP) does not have any impact on the results.



At low liquidity in the non-PvP case, the inlet coupling is lost in the internal process of congestions and cascades, and the output settlement flows of the two systems are uncorrelated.



At low liquidity and under the PvP constraint, the inlet coupling is lost in the internal process of congestions and cascades. However the PvP constraint ensures both legs of the FX transactions will settle simultaneously or never. The queue building and release processes of the two systems will therefore be correlated, as congestion in one system (preventing some FX legs to settle) will prevent the FX trades in the other system to settle as well. Conversely, a release of FX legs in a system will trigger a similar release in the other system, potentially leading to a massive cascade of settlements. The degree of coupling between the two systems can therefore be much more important than in the high liquidity case.

Fig 5: Structure of the participation in the model

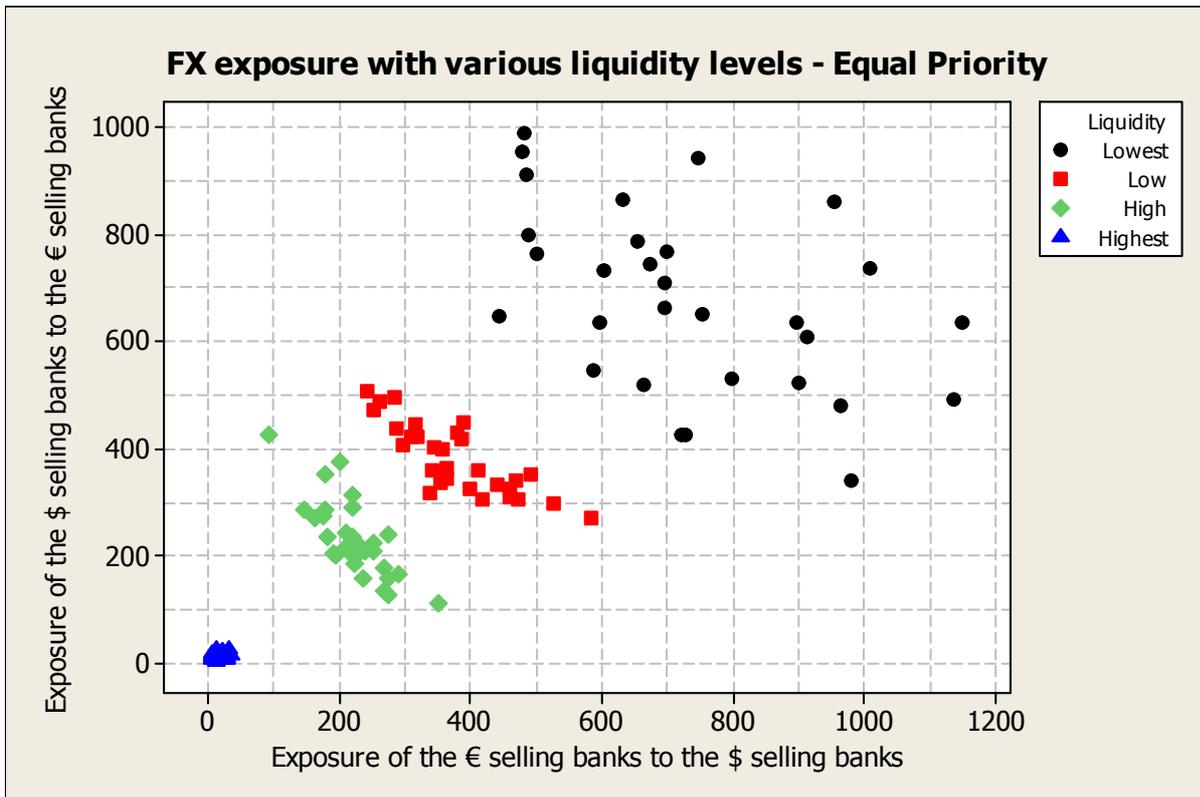


Fig 6: Gross exposures between the € selling banks and the \$ selling banks, when both RTGSs have the same level of liquidity, with a normal priority for FX payments, with a high level

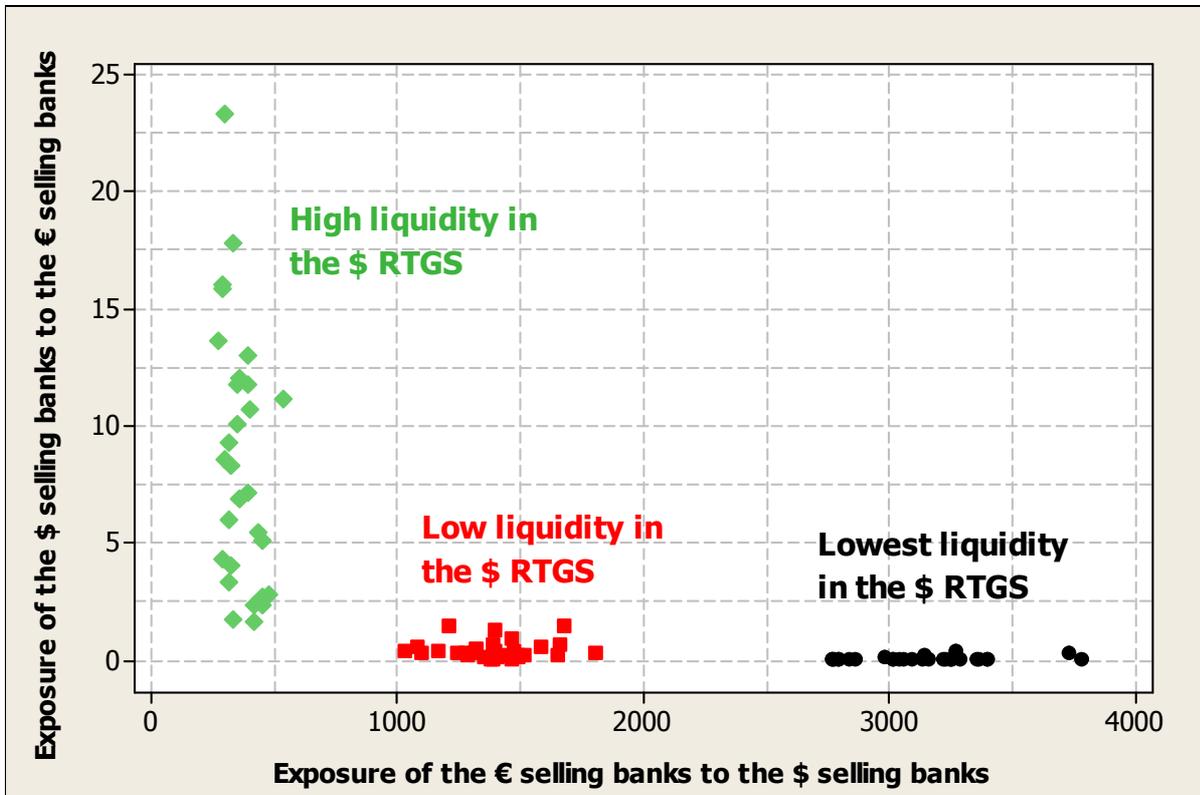


Fig 7: Influence of the liquidity level in RTGS \$ on the total gross exposures arising between the € selling banks and the \$ selling banks in the non-PvP case, with a high level of FX activity, for a constant high level of liquidity in RTGS €

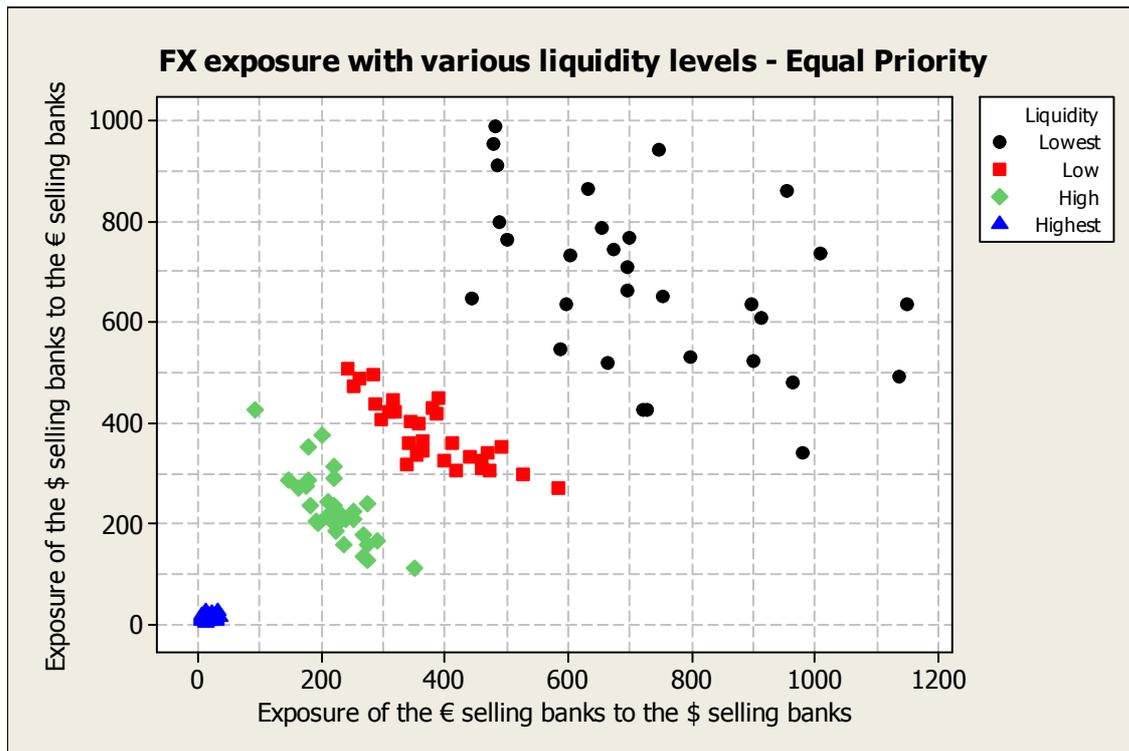


Fig 8.1: Gross exposures between the € selling banks and the \$ selling banks, when both RTGSs have the same level of liquidity, with a **normal priority for FX payments**

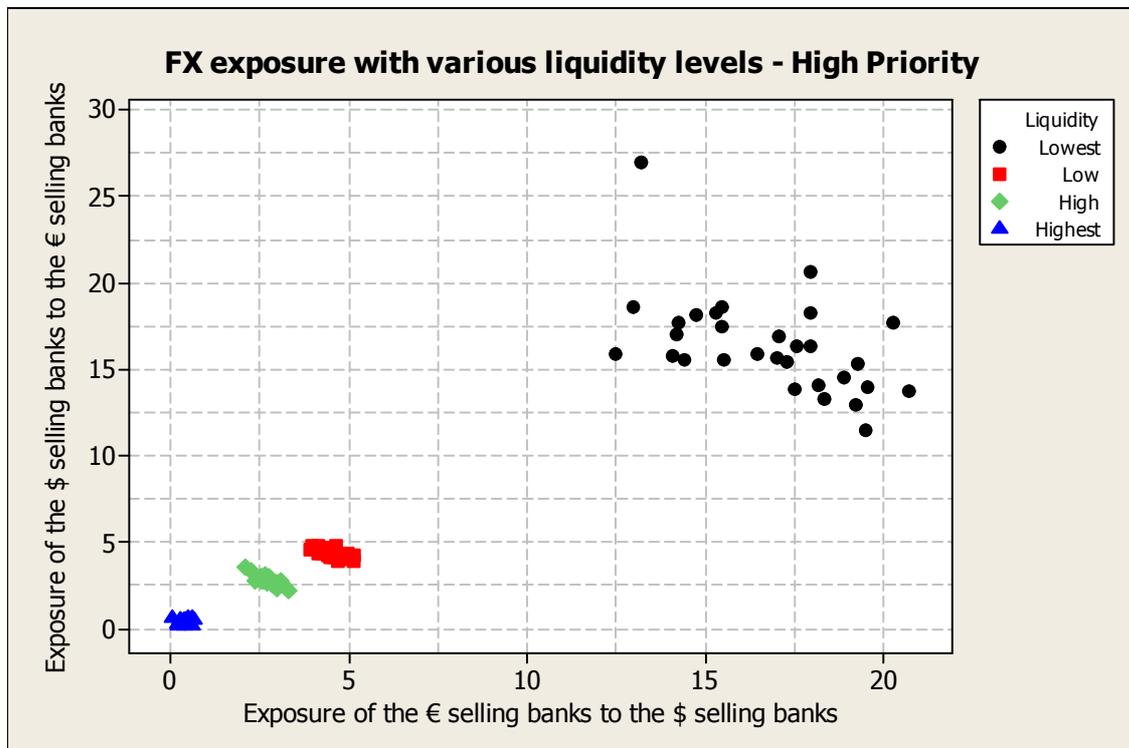


Fig 8.2: Gross exposures between the € selling banks and the \$ selling banks, when both RTGSs have the same level of liquidity, with a **high priority for FX payments**

Box 8: Influence of the relative priority of the FX payments with regard to the local payments on the total gross exposure arising between the € selling banks and the \$ selling banks, when both RTGSs have the same level of liquidity in the non-PvP case, with a high level of FX activity

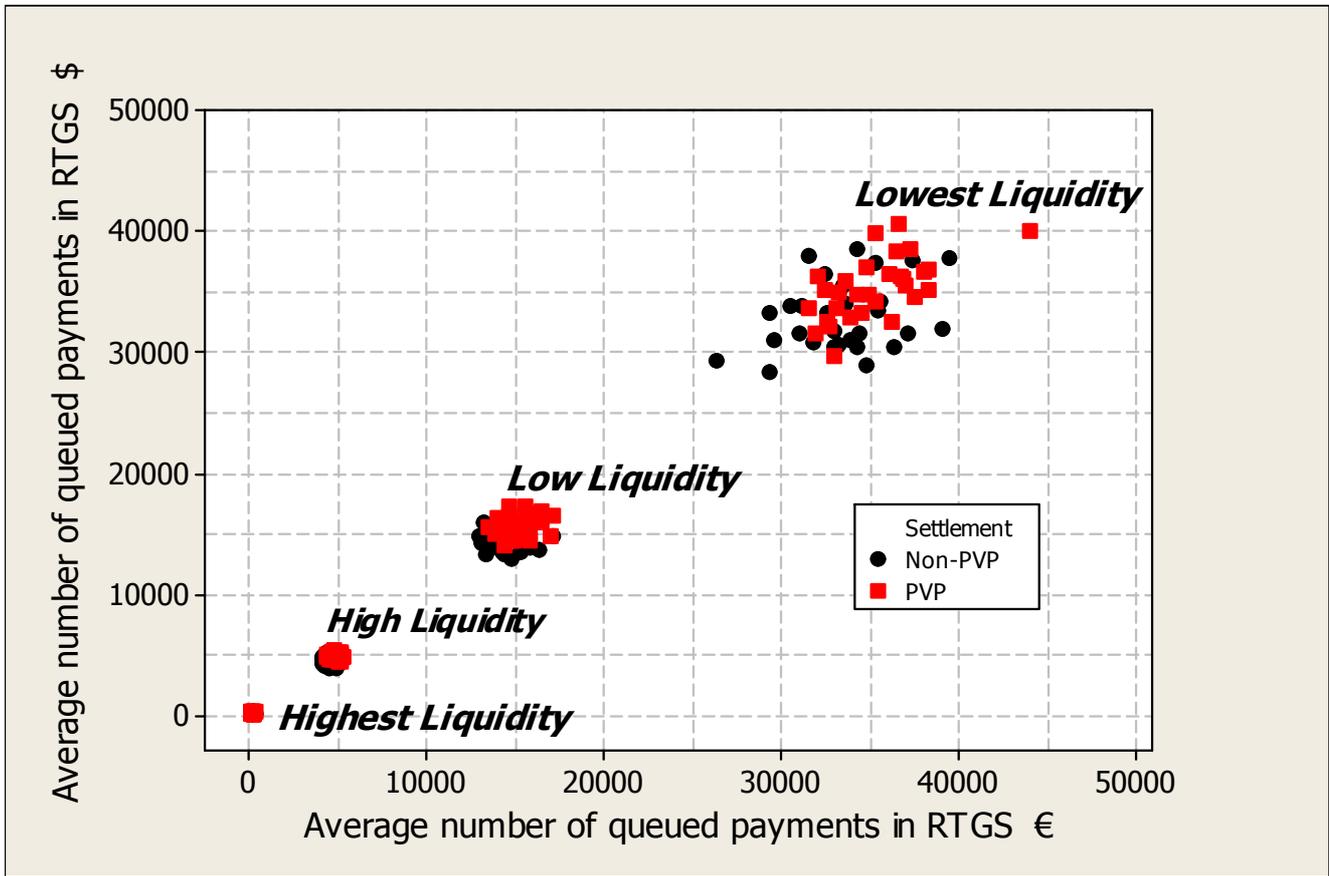


Fig 9: Influence of the PVP mechanism on the average queues in both RTGSs, when both RTGSs have the same level of liquidity, for various levels of liquidity

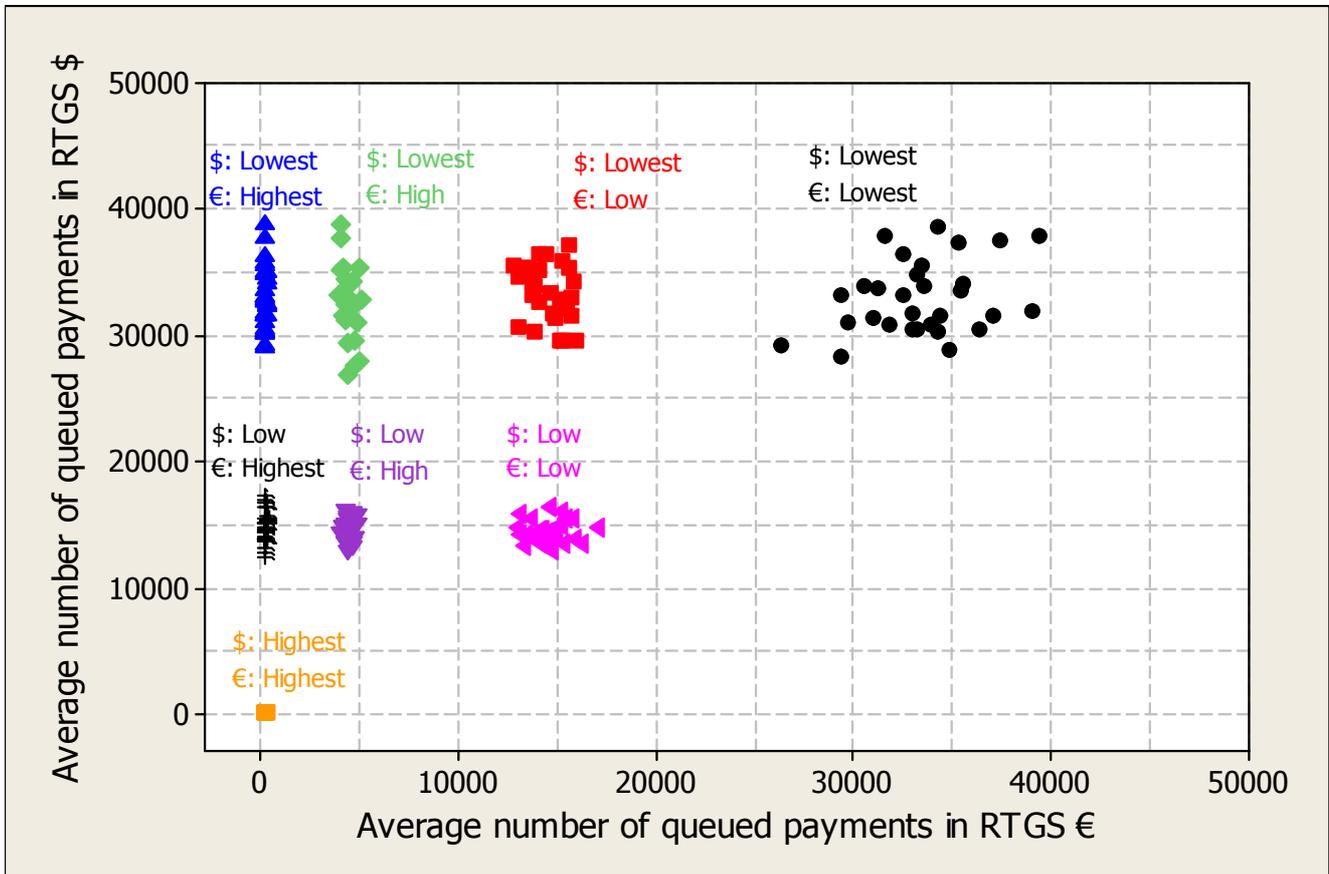


Fig 10: Average number of queued payments in both RTGSs, in the non-PvP case, for various levels of liquidity in each RTGS, and a high level of FX activity.

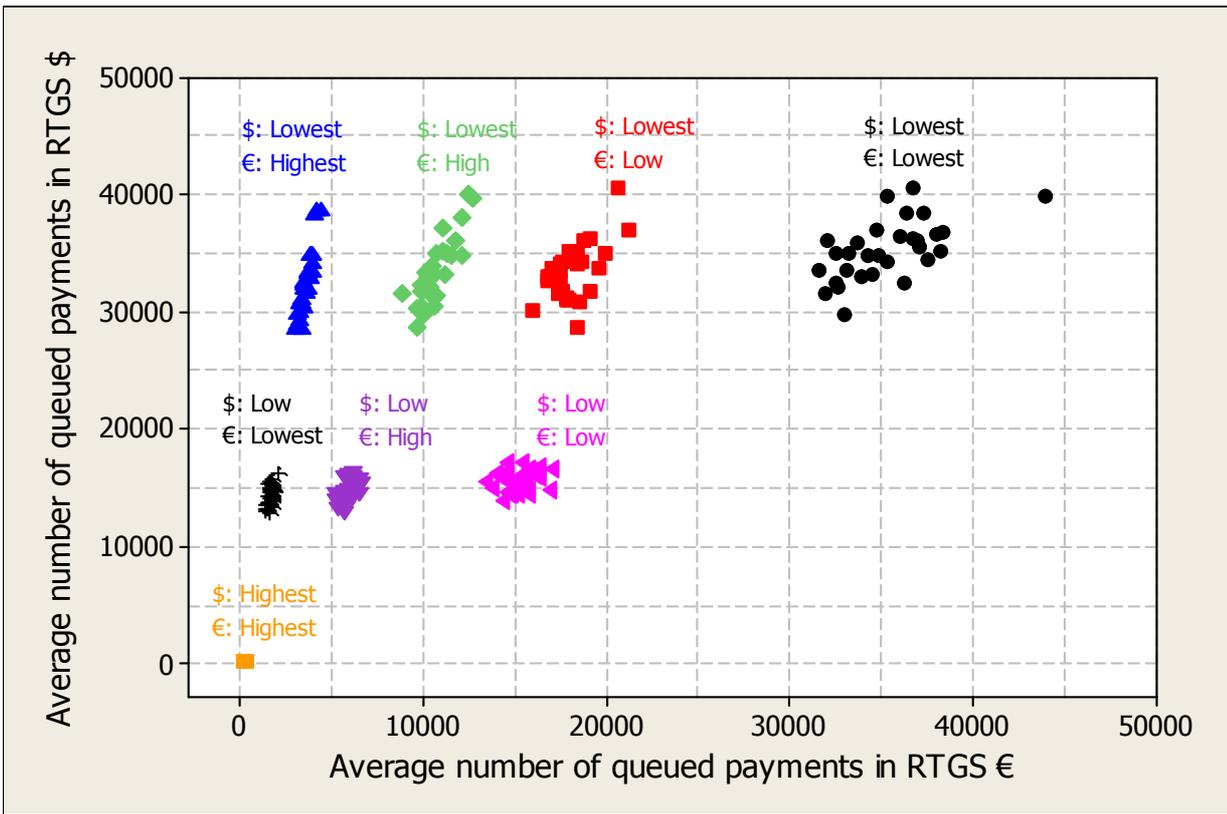


Fig 11.1: Average number of queued payments in both RTGSs, in the PvP case, for various levels of liquidity in each RTGS, and a **high level of FX activity**

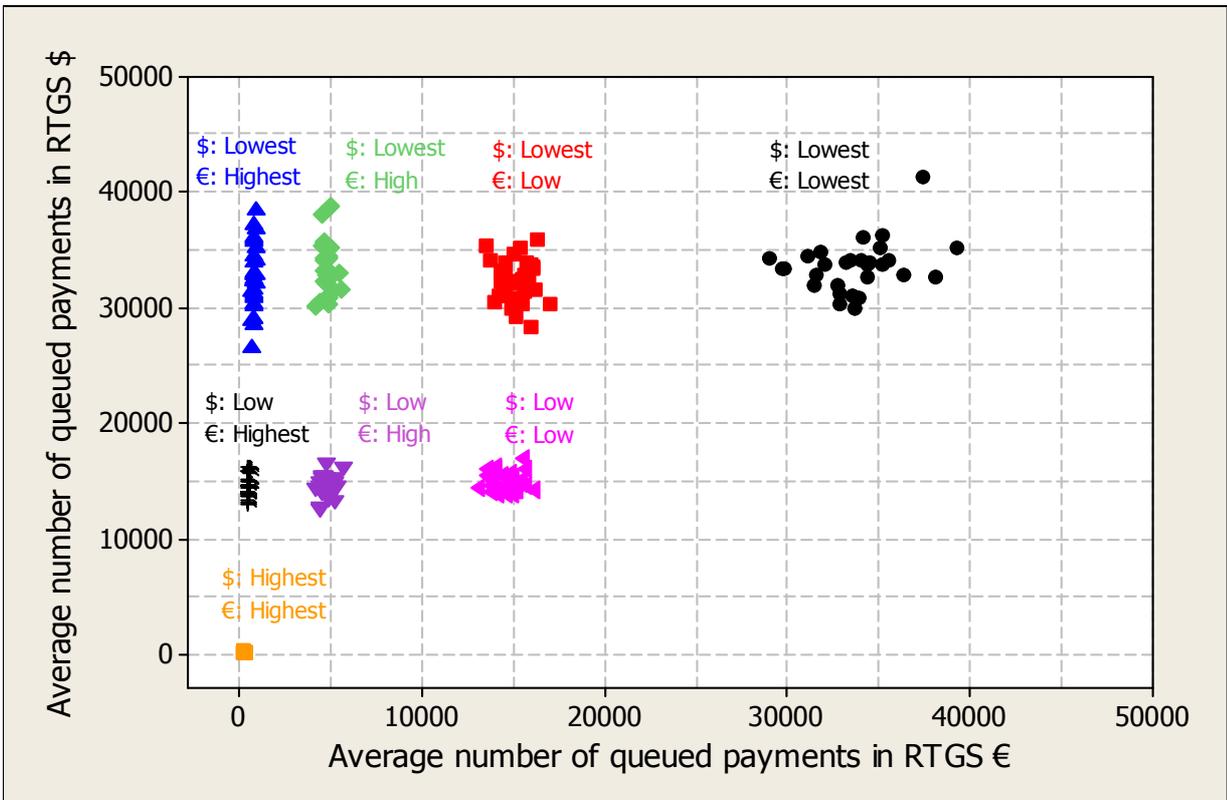


Fig 11.2: Average number of queued payments in both RTGSs, in the PvP case, for various levels of liquidity in each RTGS, and a **low level of FX activity**

Box 11: Influence of the level of FX activity on the average number of queued payments in the two RTGSs, in the PvP case, with a normal priority for FX payments

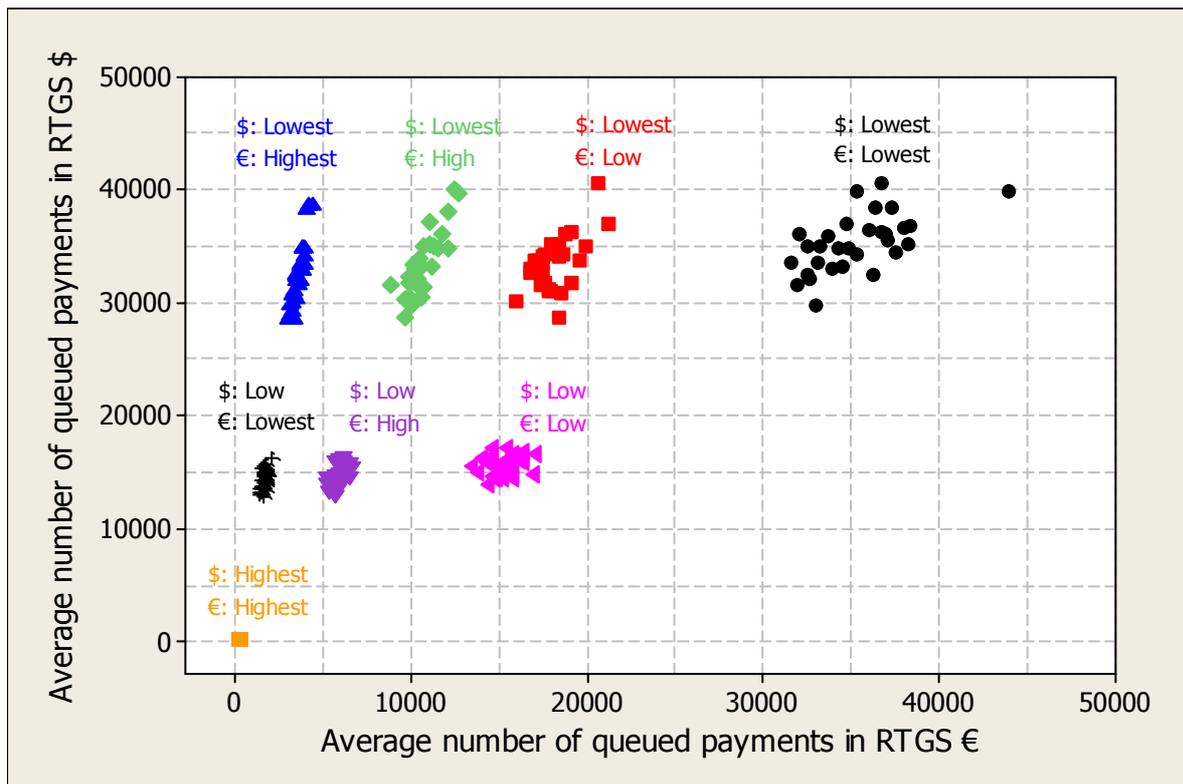


Fig 12.1: Average number of queued payments in both RTGSs, in the PvP case, for various levels of liquidity in each RTGS, and a **normal priority for FX payments**

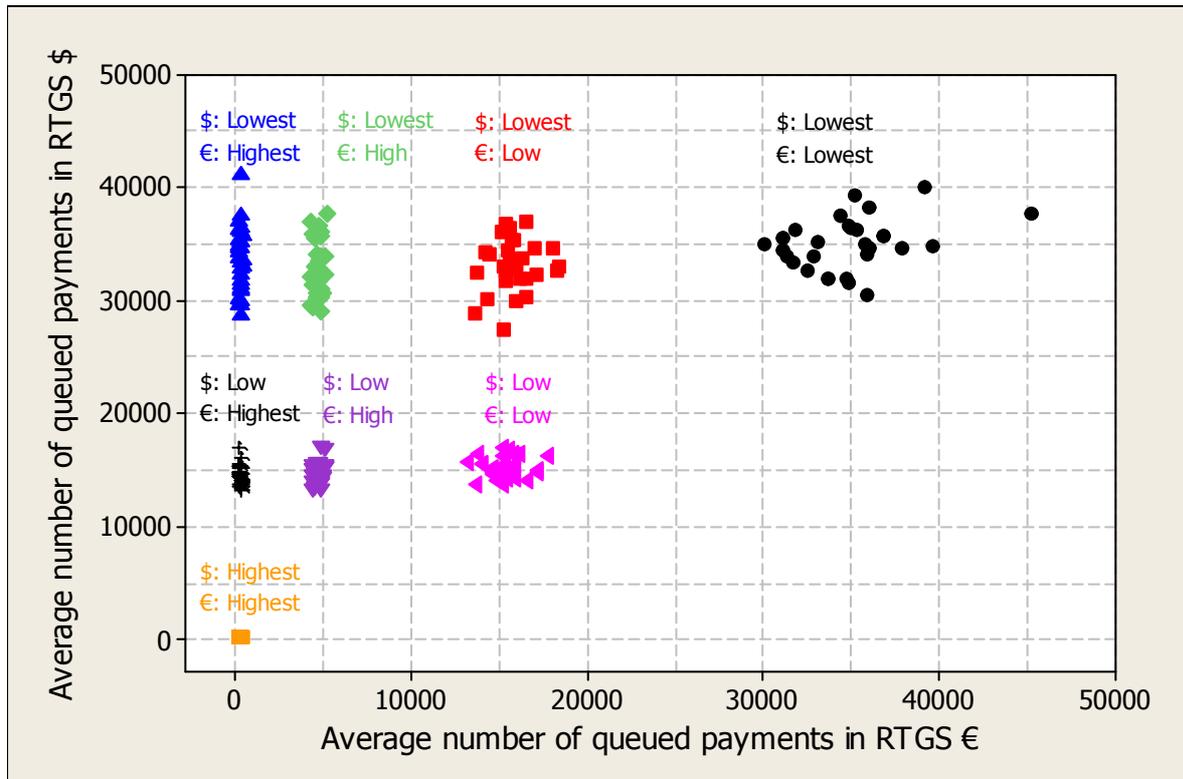


Fig 12.2: Average number of queued payments in both RTGSs, in the PvP case, for various levels of liquidity in each RTGS, and a **high priority for FX payments**

Box 12: Influence of the relative priority of the FX payments with regard to the local payments on the average level of queuing in the two RTGSs, in the PvP case, for a high level of FX activity